

**Calculus
2003
AP
Prep

Begin
Spring
Break**

Daily work is due *before* school on the day after
it is assigned!

Absolutely no late work will be considered.

There are three multiple choice packets to be done during Spring Break.
For each packet: Do the problems in order. For each problem, jot down a
brief rationale for what you did or the reason you selected which ever
answer you did. Bubble in the scantron. You will be graded on the score
correct on the scantron. Use a calculator *only* when absolutely necessary.
Include that you needed a calculator in your comment about the problem.

Turn in the Scantrons *only* before school on Monday, April 21!

1, 2
Multiple Choice # 1 No Calc

3, 4
Multiple Choice #2 No Calc

#5, 6, 7, 8
Multiple Choice #5
Calc *only* if necessary

#9, 10, 11 AP 2002 Calc on 1, 2, 3 only 4 / 21	#12, 13, 14 AP 2001 Calc on 1, 2, 3 only problem 2: function should read: $P(t) = 20 + 10te^{(-\frac{t}{3})}$ 4 / 22	# 15, 16, 17 AP 2000 Calc on 1, 2, 3 only problem 1 reads: $y = e^{-x}$ It should read $y = e^{-x^2}$ 4 / 23	# 18, 19, 20 AP 1999 Calc on 1, 2, 3 only 4 / 24	# 21, 22, 23 AP 1998 Calc use OK 4 / 25
# 24, 25, 25 AP 1997 Calc use OK 4 / 28	# 26, 27, 28 AP 1996 Calc on 1, 2, 3 only 4 / 29	# 29, 30, 31 AP 1995 Calc use OK 4 / 30	# 32, 33, 34 AP 1994 Calc use OK 5 / 1	 5 / 2
Individual Review Questions and Personal Preparation 5 / 5 No Chat! 5 / 6			Advanced Placement Calculus AB Exam 7:45 AM Library 5 / 8	5 / 9

Advanced Placement Calculus AB

Year	5	4	3	2	1	T	Ave
03	15	12	4	1	1	33	4.18
02	15	5	2	2		24	4.38
01	13	8	5	1		27	4.22
00	8	4	6	3	1	22	3.68
99	8	3	2			13	4.46
98	8	4	3	1		16	4.19
97	10	5	1	4	1	21	3.90
96	9	3	6	1		19	4.05
95	6	4	5			15	4.07
94	9	8	1		1	19	4.26
93	7	5	7	2		21	3.81
92	4	7	3		2	16	3.69
91	3	7	11	4		25	3.36
90	8	3	1	1		13	4.38
89	5	5	7	1		18	3.78
88	6	5				11	4.54
87	4	6	3	1		14	3.21
86	12	7	2	3	1	25	4.00
85	7	3	9	5	1	25	3.40
84	6	6	5			17	4.06
83	5	9	6	4	1	25	3.38
82	5	6	1	1		13	4.15
81	7	5	4	2		18	3.94
80	3	7	3			13	4.0
Totals:	183	137	97	37	9	463	3.97

SD

37

$$Y = 0.175x + 3.74$$

$$r^2 = .34$$

$$r = .12$$

24/15

Do not settle for second best

39.5

29.6

20.9

8.0

1.9

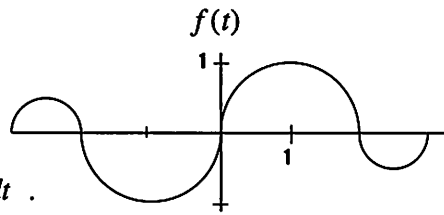
21.0

19.3



The graph of the function f on the right consists of four semicircles.

Given: $g(x) = \int_0^x f(t) dt$, $h(x) = \int_1^x f(t) dt$ and $j(x) = \int_0^{2x} f(t) dt$.



- I
 - a) Consider a circle of radius one-half. What is the area of its semicircle?
 - b) Consider a circle of radius one. What is the area of its semicircle?
 - c) Is the function $f(t)$ odd, even, or neither?

- II
 - a) How does the answer to part (Ic) effect $g(x)$?
 - b) Find: $g(0)$, $g(2)$, $g(3)$, $g(-2)$, $g(-3)$
 - c) Make a number line for $g'(x)$. Mark zeros as well as indicate positive and negative intervals.
 - d) Specifically state the intervals when $g(x)$ is increasing; decreasing.
 - e) Determine the *coordinates* of all extrema of $g(x)$. Justify, of course.
 - f) Determine the *coordinates* of all Points of Inflection of $g(x)$.
 - g) Sketch $g(x)$.
 - h) For what values of x is $g(x)$ non-negative.

- III
 - a) What is the domain of $h(x)$?
 - b) Compute: $h(1), h(0), h(2), h(3), h(-2), h(-3), h(-1)$
 - c) Draw a $h'(x)$ number line. Indicate $+$, $-$ as appropriate.
 - d) Compute *coordinates* of extrema. Justify, of course.
 - e) Draw a $h''(x)$ number line. Indicate $+$, $-$ as appropriate.
 - f) Compute *coordinates* of PI.
 - g) Sketch $h(x)$.

- IV
 - a) What is the domain of $j(x)$?
 - b) As in section III, compute the significant values of $j(x)$.
 - c) Draw $j'(x)$ number line, Indicate $+$, $-$ as appropriate.
 - d) Compute *coordinates* of extrema. Justify, of course.
 - e) Draw $j''(x)$ number line, Indicate $+$, $-$ as appropriate.
 - f) Compute *coordinates* of PI.
 - g) Sketch $j(x)$.

* Ask your parents about Ward Bond's famous words.

I a) $\frac{\pi}{8}$

I b) $\frac{\pi}{2}$

I c) f is an odd function because the function reflects through the origin.
For example, $(1, 1)$ reflects to $(-1, -1)$.

II a) g will be an even function so $g(a) = g(-a)$.

II b) $g(0) = 0$ because $g(0) = \int_0^0 f(t) dt$.

$$g(2) = \int_0^2 f(t) dt \text{ which is } \frac{\pi}{2} \text{ (from part b) so } g(-2) = g(2) = \int_0^2 f(t) dt = \frac{\pi}{2}$$

$$g(3) = \int_0^3 f(t) dt = \frac{\pi}{2} - \frac{\pi}{8} = \frac{3\pi}{8} \text{ subtracting the smaller semicircle from the larger so}$$

$$g(-3) = g(3) = \int_0^3 f(t) dt = \frac{\pi}{2} - \frac{\pi}{8} = \frac{3\pi}{8}.$$

II c) $\frac{1+0-0+0-1}{-3-2 \quad 0 \quad 2 \quad 3}$

II d) $g'(x) > 0$ for $-3 < x < -2$ and for $0 < x < 2$ thus g is increasing in these intervals.
 $g'(x) < 0$ for $-2 < x < 0$ and for $2 < x < 3$ thus g is decreasing in these intervals.

II e) $g(-3) = g(3) = \frac{3\pi}{8}$. The left end point is a minimum because g increases from it and the right end point

is a minimum because g decreases to it. coordinates: $\left(3, \frac{3\pi}{8}\right)$ and $\left(-3, \frac{3\pi}{8}\right)$

$g(-2) = g(2) = \frac{\pi}{2}$ are maximums by $f'(x)$ test. coordinates: $\left(2, \frac{\pi}{2}\right)$ and $\left(-2, \frac{\pi}{2}\right)$

$g(0) = 0$ by $f'(x)$. coordinates $(0, 0)$

II f) PI occur in g when $g''(x)$ changes sign. This is when $f'(x)$ changes sign. From the our graph, this is when f changes from increasing to decreasing (or vice versa).

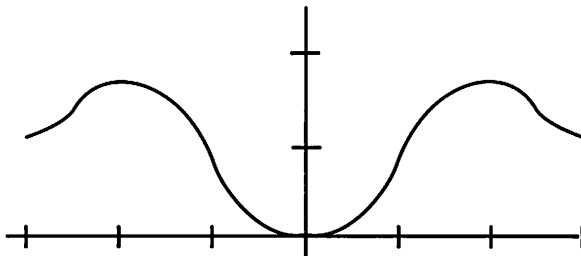
PI occur when x is an element of the set: $\{-2.5, -1, 1, 2.5\}$. Note: $\frac{\pi}{2} \cdot \frac{1}{2} = \frac{\pi}{4}$ and $\frac{7\pi}{16} = \frac{\pi}{2} - \frac{\pi}{16}$

Coordinates: $\left(-2.5, \frac{7\pi}{16}\right), \left(-1, \frac{\pi}{4}\right), \left(1, \frac{\pi}{4}\right), \left(2.5, \frac{7\pi}{16}\right)$

$\frac{\text{up}}{+} \frac{\text{dn}}{0} \frac{\text{up}}{+} \frac{\text{dn}}{0} \frac{\text{up}}{+}$
 $\frac{1}{-3} \frac{0}{-2.5} \frac{1}{-2} \frac{0}{-1} \frac{1}{0} \frac{0}{1} \frac{1}{2} \frac{0}{2.5} \frac{1}{3}$

Note: $g''(x)$ DNE for $x \in \{-2, 0, 2\}$ because the tangent line to the graph of f becomes vertical.

II g)



II h) g is non-negative for all $x \in [-3, 3]$.

III a) $D_h x \in [-3, 3]$ There is no change because upper bound is x (rather than $x+1$, for example)

$$\text{III b) } h(1) = \int_1^1 f(t) dt = 0 \quad (1, 0) \quad h(0) = \int_1^0 f(t) dt = - \int_0^1 f(t) dt = -\frac{\pi}{4} \quad \left(0, -\frac{\pi}{4}\right)$$

$$h(2) = \int_1^2 f(t) dt = \frac{\pi}{4} \quad \left(2, \frac{\pi}{4}\right) \quad h(3) = \int_1^3 f(t) dt = \frac{\pi}{4} - \frac{\pi}{8} = \frac{\pi}{8} \quad \left(3, \frac{\pi}{8}\right)$$

$$h(-1) = \frac{\pi}{4} - \frac{\pi}{4} = 0 \quad (-1, 0) \quad h(-2) = -\left(-\frac{\pi}{4}\right) = \frac{\pi}{4} \quad \left(-2, \frac{\pi}{4}\right)$$

$$h(-3) = \text{this is } h(-2) - \frac{\pi}{4} = \frac{\pi}{4} - \frac{\pi}{8} = \frac{\pi}{8} \quad \left(-3, \frac{\pi}{8}\right)$$

III c) This is the same as it was for II c because $h'(x) = f(x)$ just as $g'(x) = f(x)$.

$$\frac{0+0-0-0+0-0}{-3-2-0-2-3}$$

III d) $\left(-3, \frac{\pi}{8}\right)$ min. end pt increasing from it. $\left(-2, \frac{\pi}{4}\right)$ max $f'(x)$ test

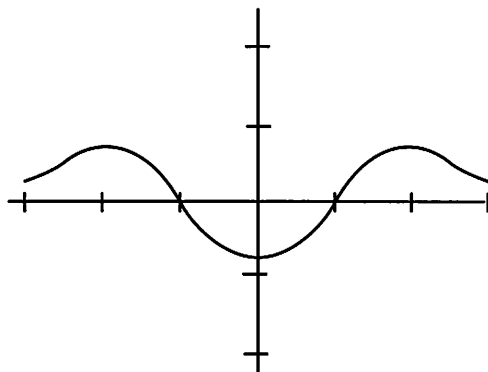
$\left(0, -\frac{\pi}{4}\right)$ min $f'(x)$ test $\left(2, \frac{\pi}{4}\right)$ max $f'(x)$ test $\left(3, \frac{\pi}{8}\right)$ min end pt decrease to it.

III e) $h''(x)$ $\frac{\text{up} \quad \text{dn} \quad \text{up} \quad \text{dn} \quad \text{up}}{\frac{1+0-0-0+0-0}{-3-2.5-2-1-0-1-2-2.5-3}}$ Note: This is exactly the same as $g''(x)$ (II f)

III f) As in part II f, the x values of the PI are the same.

$$-\frac{\pi}{4} + \frac{\pi}{2} - \frac{\pi}{16} = \frac{3\pi}{16} \quad \left(-\frac{5}{2}, \frac{3\pi}{16}\right) \quad \frac{\pi}{4} - \frac{\pi}{16} = \frac{3\pi}{16} \quad \left(\frac{5}{2}, \frac{3\pi}{16}\right) \quad (-1, 0), (1, 0)$$

III g) $h(x)$:



IV a) $-3 \leq t \leq 3$ so

$$-3 \leq 2x \leq 3 \quad D_j \quad x \in \left[-\frac{3}{2}, \frac{3}{2}\right]$$

$$-\frac{3}{2} \leq x \leq \frac{3}{2}$$

IV b) The $2x$ just changes the period of the function much in the same manner as $\sin x$ compared to $\sin 2x$.

$$j(0) = 0, \quad j(-1) = j(1) = \int_0^{2 \cdot 1 = 2} f(t) dt = \frac{\pi}{2}, \quad j\left(-\frac{3}{2}\right) = j\left(\frac{3}{2}\right) = \int_0^{2 \cdot \frac{3}{2} = 3} f(t) dt = \frac{3\pi}{8}$$

$$j\left(-\frac{1}{2}\right) = j\left(\frac{1}{2}\right) = \int_0^{2 \cdot \frac{1}{2} = 1} f(t) dt = \frac{\pi}{4} \quad j\left(-\frac{5}{4}\right) = j\left(\frac{5}{4}\right) = \int_0^{2 \cdot \frac{5}{4} = \frac{5}{2}} f(t) dt = \frac{\pi}{2} - \frac{\pi}{16} = \frac{7\pi}{16}$$

IV c) $j'(x)$ $\begin{array}{cccccc} \text{[} & + & 0 & - & 0 & + & 0 & - & \text{]} \\ -\frac{3}{2} & -1 & \underbrace{0} & \frac{1}{4} & 1 & \frac{3}{2} \end{array}$

$j'\left(\frac{1}{4}\right)$ would be the slope of $f(t)$ at $x = \frac{1}{2}$.

From the original picture of f , f is clearly increasing. The $+$, $-$ signs alternate from there.

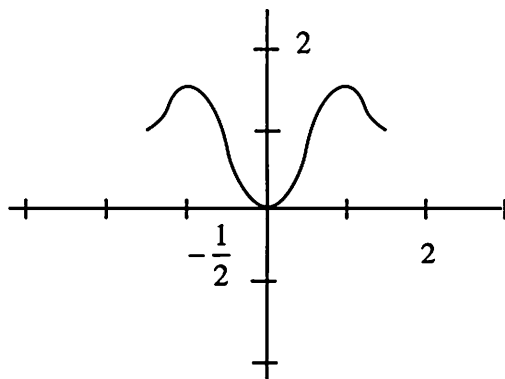
IV d) $\left(-\frac{3}{2}, \frac{3\pi}{8}\right)$ is min. end pt, increasing from it $\left(-1, \frac{\pi}{2}\right)$ is max. f' test. $(0, 0)$ min f' test.

$\left(1, \frac{\pi}{2}\right)$ max, f' test. $\left(\frac{3}{2}, \frac{3\pi}{8}\right)$ is min. end pt. decreasing to it.

IV e) $\begin{array}{cccccc} \text{up} & \text{dn} & & \text{up} & \text{dn} & \text{up} \\ \text{[} & + & 0 & - & 0 & + & 0 & - & 0 & + & \text{]} \\ -\frac{3}{2} & -\frac{5}{4} & -1 & -\frac{1}{2} & \underbrace{0} & \frac{1}{2} & 1 & \frac{5}{4} & \frac{3}{2} \end{array}$

IV f) The coordinates are listed in IV b.

IV g)



Conic Sections

Point

$$(x, y) \quad (r, \theta)$$

lines

$$y = mx + b$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Locus of pts between two pts,

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

⊥ lines $m_1 = -\frac{1}{m_2}$

Circles

$$(x - h)^2 + (y - k)^2 = r^2$$

center (h, k) radius r .

Locus of pts equidistant from one pt.

Parabola

$$y = \frac{1}{4p}(x - h)^2 + k$$

if $p > 0$ opens up

vertex (h, k)

$$x = \frac{1}{4p}(y - k)^2 + h$$

if $p > 0$ opens right

p = dist from vertex focus or V to direct

Locus of pts equidistant from a PT and a line

ellipse

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$

$$b^2 + c^2 = a^2$$

$$c^2 = a^2 - b^2$$

Locus of PTS such that $f_1P + Pf_2 = 2a$ (major axis)

hyperbola

$$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$$

if $x = h$ then we have asympt. (neg = -1) so this opens left & right and not up & down.

Locus of pts such that $f_1P - f_2P = 2a$ (transverse axis) other is conjugate

Not allowed to use a pencil after 6pm

Book over — Remember when you learned
the topic

Thumbs Trig notebook, analysis notebook
and finally Calc notebook
re read solutions to last calc
ex4m.

89-6.

EAT Normal breakfast

arrive at Library 8⁵⁰ after bathroom

Stop. Don't Bring Books, Bring
several pencils w/ food mask,
calculator, watch.

multiple choice

- ① go through with 1st results. make
pb hm # of questionable pb hms.
- ② as time allows, try to go questionable
problems in another manner or
make a ~~conclusion~~ that should also
be true and test it.
- ③ as time allows — try to see ~~where~~ reason
other choices are incorrect

	EXAM CH 9 R/C Pg 362 - 366 # 1 Pg 366 1 - 3 # 2	Pg 366 4, 7, 8, 11-13, 16 # 2 R/C Pg 367 - 371 # 3	Pg 371 -372 1, 3, 4, 5 # 4 Important Lecture	Pg 372 9, 10, 15, 16 # 5
	3/23	3/24	3/25	3/26
Pg 373 1 - 5, 8, 11, 12, 19 # 6	Pg 374 21, 31 - 35 # 7 R/C Pg 375 - 379 # 8	Pg 379 1 - 8, 10 - 12 # 9 Important Lecture	Pg 379 9, 16, 19 # 10	Pg 391 1 - 17, 22, 27 # 11 "Glaxia"
3/29	3/30	4/31	4/1	4/2
Pg 384 1 - 7, 9, 12 # 12 board Problems # 13 R/C Pg 397 - 401 # 14 Pg 414 10, 14, 15 # 25	Pg 402 1 - 7, 8 - 11, 13, 14 # 15	Pg 410 1 - 7 # 16 Important Lecture Minimum Day	Pg 410 9, 10, 12, 13 # 17 Minimum Day Parent Conferences	"Get Caught up" day maybe begin holiday HW. We have NO time to waste!
4/5	4/6	4/7	4/8	4/9
During Easter: Re-Read pages 913 - 923. Write out a careful, complete outline of everything there is to know about conic sec- tions. # 18, 19 R/C Pg 455 - 457 Write out a careful, complete outline of ev- erything there is to know about Newton's method of solving equa- tions. # 20, 21 Then do Pg 458 1, 2, 5, 7, 10 # 22 R/C pg 411-414 # 23 Do Pg 414 1 - 6, 8 # 24 AP Calculus AB Chapter 10 1993 Don't give up now! This is the Last Homework Sked!				
Pg 414 10, 14, 15 # 25	Pg 420-421 1 - 8 # 26 Important Lecture	Pg 421 22, 24, 25 # 27	Pg 427 - 428 1, 2, 3, 19, 20, 25, 26 # 28 ReWrite ENTIRE "Summary" # 29	GQ Pg 432 1, 3, 9 - 11 RE Pg 432-436 1 - 4, 6 - 8, 12 # 30
4/19	4/20	4/21	4/22	4/23
Pg 432-436 17, 25, 28, 30, 32, 39-48, 50, 51, 53, 54, 61 # 31, 32, 33	Review	Exam Ch 10 Begin AP Prep Do Booklets #1, and # 2 ! 4/28		
4/26	4/27			

① Newton's method: $x_{N+1} = x_N - \frac{f(x_N)}{f'(x_N)}$

Given $f(x)$, $x_2 = \frac{3}{2}$

$$f(x) = 4 - x^2$$

$$f'(x) = -2x$$

Determine $x_4 = 2.0016$

$$x_3 = 2.083$$

Defobser like Phlan I on last EXAM.

$\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$ is usually used to prove $\lim_{x \rightarrow a} f(x) = L$.

$\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$ not convenient usually when involving absolute values.

ALSO f.m. Piece meal fct's, also $\lim_{x \rightarrow 0} \frac{\sin x}{x}$ etc.

Reducible Domain Rest = holes in graph

Non Reducible = Vertical asymptotes

$\lim_{x \rightarrow \pm \infty} f(x) = L$ usually = horizontal asymptote.

Typical types

If $f(x) = \dots$ what can be said of $f(x)$?

or If $f(x) = \dots$ what can be said of $f(x)$?

etc.

or If $f'(x) = \dots$

eg. for which of the following is $f(x) = f(|x|)$?

a) $f(x) = x^3$

b) $f(x) = x^2 - 4x$

* c) $f(x) = x^4 - x^2$

d) $f(x) = 4 - x$

e) $f(x) = x$

must be even!
fctn!

for which of the following $f(x) = |f(x)|$

a) $f(x) = x^2 - 4$

b) $f(x) = x^3$

c) $f(x) = x^4 + x^2$

* d) $f(x) = x^2 + 6$

e) $f(x) = 4x^2 - 4x$

$f \text{ must } \geq 0 \therefore d$

• Piece meal fctns!

Der may not exist at edges of interval
But will be fine on interval
Area of interval usually.

Simpson's method for approximating

$$\frac{h}{3} (x_0 + 4x_1 + 2x_2 + 4x_3 + \dots + x_n)$$

Trapezoid

$$\frac{h}{2} (x_0 + 2x_1 + 2x_2 + \dots + x_n)$$

Do Not give ~~an~~ a calculator answer if you can avoid it.

integration by parts

$$\int v du = uv - \int u dv$$

$$\int x \ln x dx$$

$$u = \ln x \quad dv = x dx$$

$$du = \frac{1}{x} dx \quad v = \frac{x^2}{2}$$

$$\frac{x^2}{2} \ln x - \int \frac{1}{2} x dx$$

$$\frac{x^2}{2} \ln x - \frac{x^2}{4} + C$$

$$\int x^2 \cos x dx$$

$$u = x^2 \quad dv = \cos x dx$$

$$du = 2x dx \quad v = \sin x$$

$$x^2 \sin x - 2 \int x \sin x dx$$

$$- 2 \int u dv = - 2 \int x \sin x dx$$

$$du = dx \quad v = -\cos x$$

$$x^2 \sin x - 2 \left[-x \cos x + \int \cos x dx \right]$$

$$x^2 \sin x + 2x \cos x - 2 \sin x + C$$

when
Given ~~if~~ $2^x = 3$ Then $x = a$

when $3^x = 2$ Then $x = b$

Find value of ab

$$2^x = 3$$

$$\log_2 a = x \Rightarrow b^x = a$$

$$\log_2 3 = a$$

$$\log_3 2 = b$$

$$\frac{\log 3}{\log 2} = a$$

$$\frac{\log 2}{\log 3} = b$$

$$ab = 1$$

$$2^a = 3$$

$$3^b = 2$$

~~$$a \log 2 = \log 3$$~~

$$a \log_2 2 = \log_2 3$$

$$a = \log_2 3$$

likewise

$$b = \log_3 2$$

89-6

$$c) 50000 \stackrel{\text{set}}{=} 10^6 (2)^{-\frac{\tau}{6}}$$

$$\frac{50000}{1000000}$$

$$\frac{1}{20} = 2^{-\frac{\tau}{6}}$$

$$-\ln 20 = -\frac{\tau}{6} \ln 2$$

$$\frac{-\ln 20}{-\ln 2} = \frac{\tau}{6}$$

$$\tau = 25.93 \text{ yrs,}$$

(5)

$$\frac{dy}{dt}$$

$$y = 600,000$$

we know $t < 6$ because at $t = 6$, 500,000 left.

(89-6)

$$600,000 = 1,000,000 \cdot 2^{-\frac{t}{6}}$$

$$\frac{3}{5} = 2^{-\frac{t}{6}}$$

$$\ln \frac{3}{5} = -\frac{t}{6} \ln 2$$

$$-.5108 = -\frac{t}{6} (.693)$$

$$-.73697 = -\frac{t}{6}$$

$$4.42 = t \quad \text{This is when we have 600,000 left}$$

$$y = 1,000,000 \cdot 2^{-\frac{t}{6}}$$

$$\frac{dy}{dt} = -100,000$$

$$\ln y = \ln 1,000,000 + \frac{t}{6} \ln 2$$

$$\frac{1}{y} \frac{dy}{dt} = -\frac{1}{6} \ln 2$$

$$\frac{dy}{dt} = (1,000,000) \cdot 2^{-\frac{t}{6}} \left(-\frac{1}{6} \ln 2 \right) = -69,329$$

$$t = 4.42$$

$$600,124.3$$

$$600,124.3$$

OR $\frac{dy}{dt} = KY$

$$= -\frac{1}{6} \ln 2 (600,000) = 69,314.7$$

$$\frac{dy}{dt} = ky$$

y = amt of oil left in well
at time t

a) Solve for y

$$\frac{dy}{y} = k dt$$

$$\int \frac{dy}{y} = \int k dt$$

$$\ln y = kt + c. \quad \text{raise } e \text{ to power}$$

$$e^{\ln y} = e^{kt+c}$$

$$y = e^{kt+c} = e^{kt} \cdot e^c = A_2 e^{kt}$$

$$\text{where } A = e^c$$

$$y = A e^{kt} \quad \text{where } A \text{ is initial amt}$$

$$\text{ie when } t=0 \quad A = \dots$$

$$y = 100000 e^{kt}$$

$$\text{we also know when } t=6 \quad y=500,000$$

$$500,000 = 1,000,000 e^{6k}$$

$$\leftarrow \frac{1}{2} = e^{6k}$$

$$(2)^{-1} = e^{6k}$$

$$(2)^{-\frac{1}{6}} = e^k \quad \text{so}$$

final equation

$$y = 4000,000 (2)^{-\frac{t}{6}}$$

b) ~~...~~

Note $-\ln 2 = 6k$

$$\text{So } k = -\frac{1}{6} \ln 2$$

to be used in

Part B

Demo Piecewise continuous - etc.

$$f(x) = \begin{cases} \frac{x^4 - 1}{x - 1} & \text{if } x < 1 \\ k & \text{if } x \geq 1 \end{cases}$$

what value of k
is $f(x)$ cont.?

$$\lim_{x \rightarrow 1^-} x^3 + x^2 + x + 1 = 4 \quad \therefore k = 4$$

~~Is~~ Does f' exist at $x = 1$?

$$\lim_{x \rightarrow 1^-} \frac{x^3 + x^2 + x + 1 - 4}{x - 1} \quad \begin{array}{r} 1 \quad 1 \quad 1 \quad -3 \\ 1 \quad 1 \quad 2 \quad 3 \quad 0 \end{array}$$

$$\frac{x^3 + x^2 + x - 3}{x - 1}$$

$$\lim_{x \rightarrow 1^-} x^2 + 2x + 3 = 6$$

$$\lim_{x \rightarrow 1^+} \frac{4 - 4}{x - 1} = 0$$

$\neq$ $\therefore$ no den. limit DNE.

Let f be the fctn $f(x) = \cos x$. If the number c satisfies the conclusion of the mean value theorem for f on $[2, 4]$ then $c =$

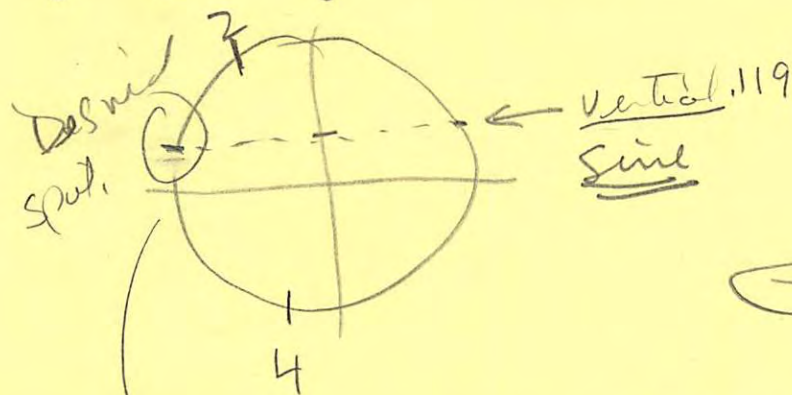
$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

Special case of Rolle's Thm
 "Slope of chord = Slope of tangent at $x = c$ "

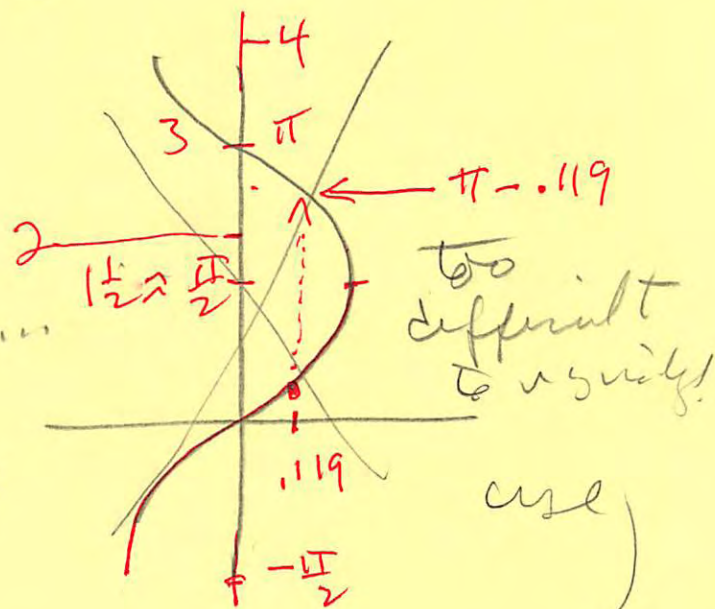
$$\frac{\cos 4 - \cos 2}{4 - 2} = -\sin c \quad \text{use calculator}$$

$$-.487418 = -\sin c$$

Pressing 2nd Sin gives $\sin c = .118748$
 $-\sin c = -.11902 \dots$



$$\pi - .1187 \dots = 3.02256$$



$$\int \tan^{-1} x \, dx$$

$$u = \tan^{-1} x$$

$$du = dx$$

$$du = \frac{1}{1+x^2} dx$$

$$v = x$$

$$x \tan^{-1} x - \int x \frac{1}{1+x^2} dx$$

$$\frac{1}{2} \int \frac{2x \, dx}{1+x^2}$$

$$x \tan^{-1} x - \frac{1}{2} \ln(1+x^2)$$

While You Were Restin'

1. Let f be the function given by $f(x) = \cos x$. If the number c satisfies the conclusion of the Mean Value Theorem (also known as the Law of the Mean) for f on the closed interval $[2, 4]$, then $c =$
- a) 0.119 b) 2.902 c) 3.023 d) 3.142 e) 3.261
2. The rate of decay of a radio active substance is proportional to the amount of the substance present at any time t . In 1840 there were 50 grams of the substance and in 1910 there were 35 grams. To the nearest gram, how many grams of the substance remain in 1990?
- a) 18 b) 20 c) 23 d) 36 e) 107
3. If $f(x) = \sqrt[5]{x^3 - 2x}$, then $f'(\sqrt{3}) =$
- a) 0.129 b) 0.139 c) 0.902 d) 0.906 e) 1.116
4. The slope of the line containing the point $(4, 0)$ and tangent to the graph of $y = e^{-x}$ is
- a) 0.50 b) 0.18 c) -0.50 d) -0.18 e) -0.07
5. If $f(x) = \ln x$, then the average rate of change of f on the interval $[2, 5]$ is
- a) 0.100 b) 0.133 c) 0.305 d) 1.155 e) 1.220
6. A 16-foot ladder is leaning against a vertical wall. The tip of the ladder is sliding down the wall at the rate of 5.6 feet per second. What is the rate of change, in radians per second, of the angle θ at the instant when the tip of the ladder is 7 feet above the ground?
- a) -6.2 b) -0.8 c) -0.4 d) -0.3 e) -0.1
7. The slope of the line tangent to the curve $y = x^{-x}$ at $x = 0.368$ is closest to the integer
- a) -6 b) -1 c) 0 d) 1 e) 6
8. What pattern did the correct answers make?

A.P. Multiple Choice #1

Mr. Mumaugh



The area of one loop of the graph of the polar equation $r = 2 \sin(3\theta)$ is given by which of the following expressions?

- a) $4 \int_0^{\frac{\pi}{3}} \sin^2(3\theta) d\theta$ b) $2 \int_0^{\frac{\pi}{3}} \sin(3\theta) d\theta$ c) $2 \int_0^{\frac{\pi}{3}} \sin^2(3\theta) d\theta$ d) $2 \int_0^{\frac{2\pi}{3}} \sin^2(3\theta) d\theta$ e) $2 \int_0^{\frac{2\pi}{3}} \sin(3\theta) d\theta$

38. Let g be the function given by $g(t) = 100 + 20 \sin\left(\frac{\pi t}{2}\right) + 10 \cos\left(\frac{\pi t}{6}\right)$

- a) 0.949 b) 2.017 c) 3.106 d) 5.965 e) 8.000

46. What is $\lim_{h \rightarrow 0} \frac{\cos\left(\frac{\pi}{2} + h\right) - \cos\left(\frac{\pi}{2}\right)}{h}$?

- a) -1 b) $-\frac{\sqrt{2}}{2}$ c) 0 d) 1 e) DNE

47. The slope of the tangent to the curve $y^3x + y^2x^2 = 6$ at $(2, 1)$ is

- a) $-\frac{3}{2}$ b) -1 c) $-\frac{5}{14}$ d) $-\frac{3}{14}$ e) 0

48. Which of the following statements about the function given by $f(x) = x^4 - 2x^3$ is true?

- a) The function has no relative extremum.
 b) The graph of the function has one PI and the function has 2 relative extrema.
 c) The graph of the function has 2 PI and the function has 1 relative extremum.
 d) The graph of the function has 2 PI and has two relative extrema.
 e) The graph of the function has 2 PI and the function has 3 relative extrema.

49. If $f(x) = \sin^2(3 - x)$, then $f'(0) =$

- a) $-2 \cos 3$ b) $-2 \sin 3 \cos 3$ c) $6 \cos 3$ d) $2 \sin 3 \cos 3$ e) $6 \sin 3 \cos 3$

50. The solution to the differential equation $\frac{dy}{dx} = \frac{x^3}{y^2}$, where $y(2) = 3$ is

- a) $y = \sqrt[3]{\frac{3}{4}x^4}$ b) $y = \sqrt[3]{\frac{3}{4}x^4} + \sqrt[3]{15}$ c) $y = \sqrt[3]{\frac{3}{4}x^4} + 15$
 d) $y = \sqrt[3]{\frac{3}{4}x^4} + 5$ e) $y = \sqrt[3]{\frac{3}{4}x^4} + 15$



A.P. Multiple Choice # 2


 $\int (x-1)\sqrt{x} \, dx =$

- a) $\frac{3}{2}\sqrt{x} - \frac{1}{\sqrt{x}} + C$ b) $\frac{3}{2}x^{\frac{3}{2}} - \frac{1}{2}x^{\frac{1}{2}} + C$ c) $\frac{1}{2}x^2 - x + C$
 d) $\frac{2}{5}x^{\frac{5}{2}} - \frac{2}{3}x^{\frac{3}{2}} + C$ e) $\frac{1}{2}x^2 + 2x^{\frac{3}{2}} - x + C$

41. The area of the region in the first quadrant between the graph of $y = x\sqrt{4-x^2}$ and the x -axis is

- a) $\frac{2}{3}\sqrt{2}$ b) $\frac{8}{3}$ c) $2\sqrt{2}$ d) $2\sqrt{3}$ e) $\frac{16}{3}$

42. Which of the following are antiderivatives of $\frac{\ln^2 x}{x}$?

I $\frac{\ln^3 x}{3}$ II $\frac{\ln^3 x}{3} + 6$ III $\frac{2\ln x - \ln^2 x}{x^2}$

- a) I only b) III only c) I and II only d) I and III only e) II and III only

46.

x	1.1	1.2	1.3	1.4
$f(x)$	4.18	4.38	4.56	4.73

Let f be a function such that $f''(x) < 0$ for all x in the closed interval $[1, 2]$, with selected values shows in the table above. Which of the following must be true about $f'(1.2)$?

- a) $f'(1.2) < 0$ b) $0 < f'(1.2) < 1.6$ c) $1.6 < f'(1.2) < 1.8$
 d) $1.8 < f'(1.2) < 2$ e) $2.0 < f'(1.2)$

47. Two particles start at the origin and move along the x -axis. For $0 \leq t \leq 10$, their respective position functions are given by $x_1 = \sin t$ and $x_2 = e^{-2t} - 1$.

For how many values of t do the particles have the same velocity?

- a) None b) 1 c) 2 d) 3 e) 4

48. If the function g is defined by $g(x) = \int_0^x \sin(t^2) \, dt$ on the closed interval $-1 \leq x \leq 3$, then g has a local minimum at $x =$

- 
 a) 0 b) 1.084 c) 1.772 d) 2.171 e) 2.507

A.P. Multiple Choice # 2

4

Oil is leaking from a tanker at the rate of $R(t) = 2,000e^{-0.2t}$ gallons per hour, where t is measured in hours. How much oil has leaked out of the tanker after 10 hours?

- a) 54 gallons b) 271 gallons c) 865 gallons d) 8,647 gallons e) 14,778 gallons

50. If f is continuous for all x , which of the floowing integrals necessarily have the same value?

I $\int_a^b f(x) dx$ II $\int_0^{b-a} f(x+a) dx$ III $\int_{a+c}^{b+c} f(x+c) dx$

- a) I and II only b) I and III only c) II and III only d) I, II, and II
e) No two necessarily have the same value.

A.P. Multiple Choice # 5

- 92) A point (x, y) is moving along a curve $y = f(x)$. At the instant when the slope of the curve is $-\frac{1}{3}$, the coordinate of the point is increasing at the rate of 5 units per second. The rate of change, in units per second, of the y -coordinate of the point is

a) $-\frac{5}{3}$ b) $-\frac{1}{3}$ c) $\frac{1}{3}$ d) $\frac{3}{5}$ e) $\frac{5}{3}$

- 93) If $\frac{dy}{dx} = y \sec^2 x$ and $y = 5$ when $x = 0$, then $y =$

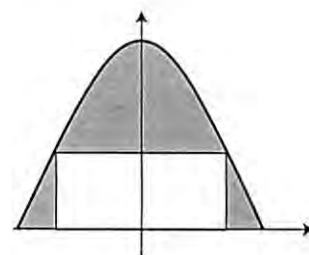
a) $e^{\tan x} + 4$ b) $e^{\tan x} + 5$ c) $5e^{\tan x}$ d) $\tan x + 5$ e) $\tan x + 5e^x$

- 94) If f is differentiable at $x = a$, which of the following could be false?

a) f is continuous at $x = a$. b) $\lim_{x \rightarrow a} f(x)$ exists. c) $\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$ exists
d) $f'(a)$ is defined e) $f''(a)$ is defined

- 95) A rectangle with one side on the x -axis has its upper vertices on the graph of $y = \cos x$ as shown on the right. What is the minimum area of the shaded region?

a) 0.799 b) 0.878 c) 1.140
d) 1.439 e) 2.000



- 96)
- | | | | | | | |
|------------------|-----|-----|-----|-----|-----|-----|
| Time (sec) | 0 | 10 | 25 | 37 | 46 | 60 |
| Rate (gal / sec) | 500 | 400 | 350 | 280 | 200 | 180 |

The table above gives the values for the rate (in gal/sec) at which water flowed into a lake, with readings taken at specific times. A right Riemann sum, with the five subintervals indicated by the data in the table, is used to estimate the total amount of water that flowed into the lake during the time period $0 \leq t \leq 60$. What is the estimate?

a) 1,910 gal b) 14,100 gal c) 16,930 gal d) 18,725 gal e) 20,520 gal

- 97) A particle moves along the x -axis so that at any time $t \geq 0$ its velocity is given by $v(t) = \ln(t + 1) - 2t + 1$. The total distance traveled by the particle from $t = 0$ to $t = 2$ is

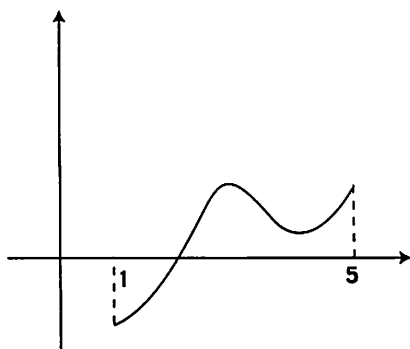
a) 0.667 b) 0.704 c) 1.540 d) 2.667 e) 2,901

A.P. Multiple Choice # 5

- 85) The line perpendicular to the tangent of the curve represented by the equation $y = x^2 + 6x + 4$ at the point $(-2, -4)$ also intersects the curve at $x =$
- a) -6 b) $-\frac{9}{2}$ c) $-\frac{7}{2}$ d) -3 E) $-\frac{1}{2}$
- 86) If $y = x + \sin(xy)$, then $\frac{dy}{dx} =$
- a) $1 + \cos(xy)$ b) $1 + y \cos(xy)$ c) $\frac{1}{1 - \cos(xy)}$ d) $\frac{1}{1 - x \cos(xy)}$ e) $\frac{1 + y \cos(xy)}{1 - x \cos(xy)}$
- 87) If $\frac{d}{dx} f(x) = g(x)$ and if $h(x) = x^2$, then $\frac{d}{dx} f(h(x)) =$
- a) $g(x^2)$ b) $2xg(x)$ c) $g'(x)$ d) $2xg(x^2)$ e) $x^2g(x^2)$
- 88) If F' is a continuous function for all real x , then $\lim_{h \rightarrow 0} \frac{1}{h} \int_a^{a+h} F'(x) dx$ is
- a) 0 b) $F(0)$ c) $F(a)$ d) $F'(0)$ e) $F'(a)$
- 89) $\int_0^\infty e^{-2t} dt$ is
- a) -1 b) $-\frac{1}{2}$ c) $\frac{1}{2}$ d) 1 e) divergent
- 90) What is the area of the closed region bounded by the curve $y = e^{2x}$ and the lines $x = 1$ and $y = 1$?
- a) $\frac{2 - e^2}{2}$ b) $\frac{e^2 - 3}{2}$ c) $\frac{3 - e^2}{2}$ d) $\frac{e^2 - 2}{2}$ e) $\frac{e^2 - 1}{2}$
- 91) Which of the following integrals gives the length of the graph of $y = \sqrt{x}$ between $x = a$ and $x = b$, where $0 < a < b$?
- a) $\int_a^b \sqrt{x^2 + x} dx$ b) $\int_a^b \sqrt{x + \sqrt{x}} dx$ c) $\int_a^b \sqrt{x + \frac{1}{2\sqrt{x}}} dx$ d) $\int_a^b \sqrt{1 + \frac{1}{2\sqrt{x}}} dx$ e) $\int_a^b \sqrt{1 + \frac{1}{4x}} dx$

A.P. Multiple Choice # 5

98)



Let f be a function whose domain is the open interval $(1, 5)$. The figure above shows the graph of $f''(x)$. Which of the following describes the relative extrema of $f'(x)$ and the points of inflection of the graph of $f(x)$?

- a) 1 relative maximum, 1 relative minimum, and no PI.
- b) 1 relative maximum, 2 relative minima, and no PI.
- c) 1 relative maximum, 1 relative minimum, and 1 PI.
- d) 1 relative maximum and 2 points of inflection.
- e) 1 relative minimum and 2 points of inflection.

If the function f is defined by $f(x) = \sqrt{x^3 + 2}$ and g is an antiderivative of f such that $g(3) = 5$, then $g(1) =$

- a) -3.268
- b) -1.585
- c) 1.732
- d) 6.585
- e) 11.585

100) Let g be the function given by $g(x) = \int_1^x 100(t^2 - 3t + 2)e^{-t^2} dt$. Which of the following statements about g must be true?

- I g is increasing on $(1, 2)$
- II g is increasing on $(2, 3)$
- III $g(3) > 0$

- a) I only
- b) II only
- c) III only
- d) II and III only
- e) I, II, and III



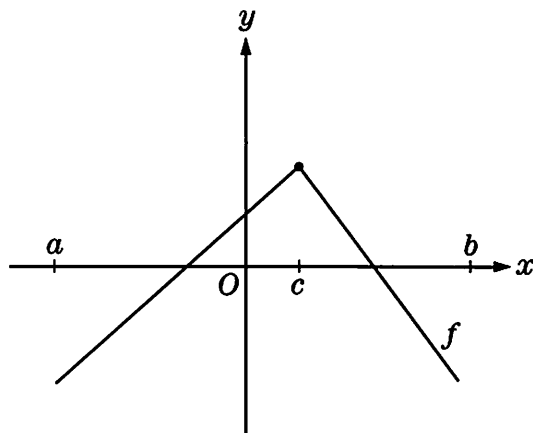
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MC #1

AP[®] Calculus AB Sample Multiple-Choice Questions

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THIS IS NOT A COMPLETE EXAMINATION.

1.



The function f , whose graph consists of two line segments, is shown above. Which of the following are true for f on the open interval (a, b) ?

- I. The domain of the derivative of f is the open interval (a, b) . *no derivative at x=c*
- II. f is continuous on the open interval (a, b) . *yes*
- III. The derivative of f is positive on the open interval (a, c) . *yes f' > 0*

- (A) I only
- (B) II only
- (C) III only
- (D) II and III only
- (E) I, II, and III

#1
46

2. What is $\lim_{h \rightarrow 0} \frac{\cos(\frac{\pi}{2} + h) - \cos(\frac{\pi}{2})}{h}$?

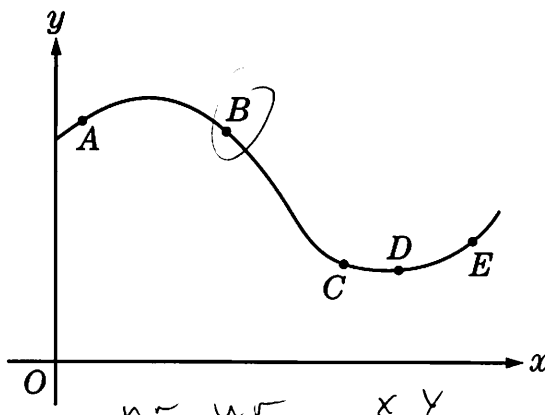
- (A) -1
- (B) $-\frac{\sqrt{2}}{2}$
- (C) 0
- (D) 1
- (E) The limit does not exist.

Def-deriv of $\cos x$ at $x = \frac{\pi}{2}$

$$-\sin x \Big|_{x=\frac{\pi}{2}} = -1$$

- 3/ At which of the five points on the graph in the figure at the right are $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ both negative?

- (A) A
(B) B ←
(C) C
(D) D
(E) E



4. The slope of the tangent to the curve $y^3x + y^2x^2 = 6$ at $(2, 1)$ is

- (A) $-\frac{3}{2}$
(B) -1
(C) $-\frac{5}{14}$
(D) $-\frac{3}{14}$
(E) 0

$$x \cdot 3y^2 \frac{dy}{dx} + y^3 + 2x^2y \frac{dy}{dx} + 2y^2x = 0$$

$$6 \frac{dy}{dx} + 1 + 8 \frac{dy}{dx} + 4 = 0$$

$$14 \frac{dy}{dx} = -5$$

$$\frac{dy}{dx} = -\frac{5}{14}$$

5. Which of the following statements about the function given by $f(x) = x^4 - 2x^3$ is true?

- (A) The function has no relative extremum.
(B) The graph of the function has one point of inflection and the function has two relative extrema.
(C) The graph of the function has two points of inflection and the function has one relative extremum.
(D) The graph of the function has two points of inflection and the function has two relative extrema.
(E) The graph of the function has two points of inflection and the function has three relative extrema.

6. If $f(x) = \sin^2(3 - x)$, then $f'(0) =$

- (A) $-2 \cos 3$
(B) $-2 \sin 3 \cos 3$
(C) $6 \cos 3$
(D) $2 \sin 3 \cos 3$
(E) $6 \sin 3 \cos 3$

$$f' = -2 \sin(3-x) \cos(3-x)$$

$$f'(0) = -2 \sin 3 \cos 3$$

7. The solution to the differential equation $\frac{dy}{dx} = \frac{x^3}{y^2}$, where $y(2) = 3$, is

- (A) $y = \sqrt[3]{\frac{3}{4}x^4}$
 (B) $y = \sqrt[3]{\frac{3}{4}x^4} + \sqrt[3]{15}$
 (C) $y = \sqrt[3]{\frac{3}{4}x^4} + 15$
 (D) $y = \sqrt[3]{\frac{3}{4}x^4} + 5$

→ (E) $y = \sqrt[3]{\frac{3}{4}x^4} + 15$

8. $\int (x-1)\sqrt{x} dx =$

- (A) $\frac{3}{2}\sqrt{x} - \frac{1}{\sqrt{x}} + C$
 (B) $\frac{2}{3}x^{\frac{3}{2}} + \frac{1}{2}x^{\frac{1}{2}} + C$
 (C) $\frac{1}{2}x^2 - x + C$

→ (D) $\frac{2}{5}x^{\frac{5}{2}} - \frac{2}{3}x^{\frac{3}{2}} + C$

(E) $\frac{1}{2}x^2 + 2x^{\frac{3}{2}} - x + C$

$$y^2 + y = x^3 dx$$

$$\frac{1}{3}y^3 = \frac{1}{4}x^4 + C$$

$$y^3 = \frac{3}{4}x^4 + C$$

$$y = \sqrt[3]{\frac{3}{4}x^4 + C}$$

$$y(2) = \sqrt[3]{\frac{3}{4}(2)^4 + C} = 3$$

$$\int x^{\frac{3}{2}} - x^{\frac{1}{2}} dx$$

$$\frac{2}{5}x^{\frac{5}{2}} - \frac{2}{3}x^{\frac{3}{2}} + C$$

$$\sqrt[3]{12+C} = 3$$

$$12+C = 27$$

$$C = 15$$

Part A Sample Multiple-Choice Questions

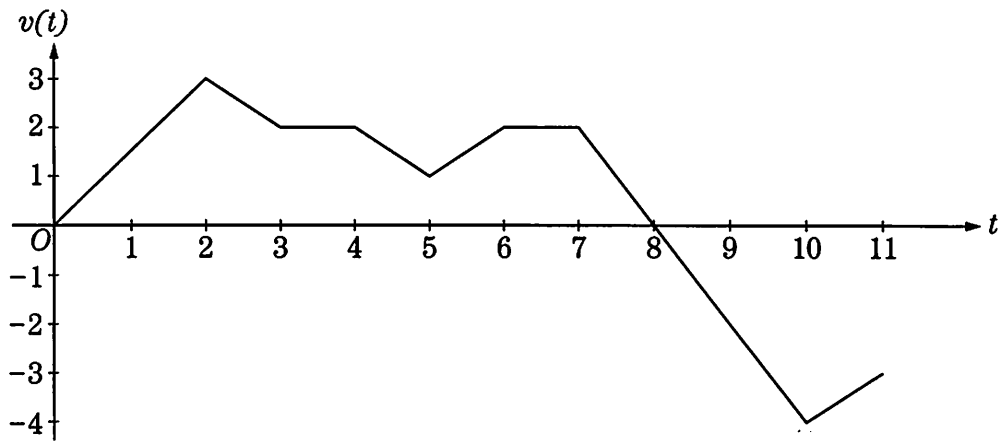
A calculator may not be used on this part of the examination.

Part A consists of 28 questions. In this section of the examination, as a correction for guessing, one-fourth of the number of questions answered incorrectly will be subtracted from the number of questions answered correctly. Following are the directions for Section I Part A and a representative set of 14 questions.

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

Questions 9 and 10 refer to the following graph and information.



A bug is crawling along a straight wire. The velocity, $v(t)$, of the bug at time t , $0 \leq t \leq 11$, is given in the graph above.

9. According to the graph, at what time t does the bug change direction?

- (A) 2
- (B) 5
- (C) 6
- (D) 8 $\Leftarrow$
- (E) 10

10. According to the graph, at what time t is the speed of the bug greatest?

- (A) 2
- (B) 5
- (C) 6
- (D) 8
- (E) 10 $\Leftarrow$

$$t = 10$$

Answers to Calculus AB Multiple-Choice Questions

Part A

- | | | |
|------|-------|-------|
| 1. D | 6. B | 11. B |
| 2. A | 7. E | 12. B |
| 3. B | 8. D | 13. C |
| 4. C | 9. D | 14. B |
| 5. C | 10. E | |

Part B

- | | | |
|--------|--------|--------|
| 15.* C | 19.* D | 22.* C |
| 16. B | 20.* E | 23.* D |
| 17. D | 21. E | 24. A |
| 18. D | | |

* Indicates a graphing calculator-active question.

14. If r is positive and increasing, for what value of r is the rate of increase of r^3 twelve times that of r ?

- (A) $\sqrt[3]{4}$
 (B) 2
 (C) $\sqrt[3]{12}$
 (D) $2\sqrt{3}$
 (E) 6

$$\frac{d(r^3)}{dr} = 12 \frac{dr}{dr}$$

$$3r^2 = 12$$

$$r^2 = 4$$

$$r = 2$$

Part B Sample Multiple-Choice Questions

A graphing calculator is required for some questions on this part of the examination.

Part B consists of 17 questions. In this section of the examination, as a correction for guessing, one-fourth of the number of questions answered incorrectly will be subtracted from the number of questions answered correctly. Following are the directions for Section I Part B and a representative set of 10 questions.

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test:

- (1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
- (2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

15. The average value of the function $f(x) = e^{-x^2}$ on the closed interval $[-1, 1]$ is

- (A) 0
 (B) 0.368
 (C) 0.747
 (D) 1
 (E) 1.494

$$\frac{1}{2} \int_{-1}^1 e^{-x^2} dx = .7468$$

16. The volume generated by revolving about the x -axis the region enclosed by the graphs of $y = 2x$ and $y = 2x^2$, for $0 \leq x \leq 1$, is

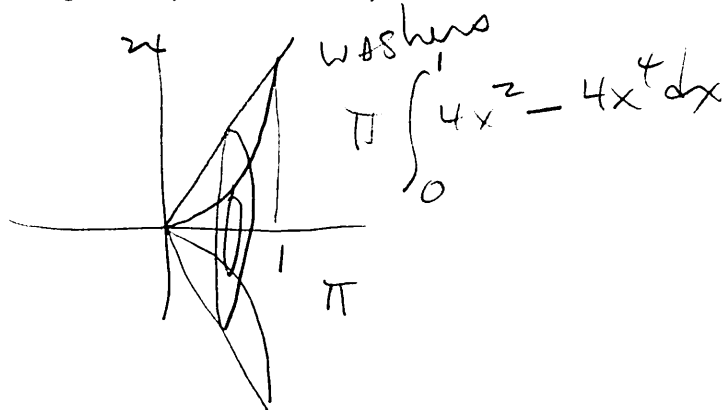
(A) $\pi \int_0^1 (2x - 2x^2)^2 dx$

→ (B) $\pi \int_0^1 (4x^2 - 4x^4) dx$

(C) $2\pi \int_0^1 x(2x - 2x^2) dx$

(D) $\pi \int_0^2 \left(\sqrt{\frac{y}{2}} - \frac{y}{2} \right)^2 dy$

(E) $\pi \int_0^2 \left(\frac{y}{2} - \frac{y^2}{2} \right) dy$



17. Let f be defined as follows, where $a \neq 0$.

$$f(x) = \begin{cases} \frac{x^2 - a^2}{x - a}, & \text{for } x \neq a, \\ 0, & \text{for } x = a. \end{cases}$$

$\lim_{x \rightarrow a} f(x) = 2a$
 $f(a) = 0$

$\left. \begin{array}{l} \lim_{x \rightarrow a} f(x) = 2a \\ f(a) = 0 \end{array} \right\} \neq \text{not cont.}$

Which of the following are true about f ?

- I. $\lim_{x \rightarrow a} f(x)$ exists. Yes $2a$
 II. $f(a)$ exists. Yes $f(a) = 0$
 III. $f(x)$ is continuous at $x = a$. No

- (A) None
 (B) I only
 (C) II only
 (D) I and II only ←
 (E) I, II, and III

$$\begin{array}{r}
 1,2 \\
 4.38 \\
 -4.18 \\
 \hline
 .2
 \end{array}$$

$$\begin{array}{r}
 .2 \\
 \hline
 .1
 \end{array}$$

$$2$$

$$\begin{array}{r}
 1\ 3 \\
 4.56 \\
 -4.18 \\
 \hline
 .38 \\
 \hline
 .2 \\
 \hline
 1
 \end{array}$$

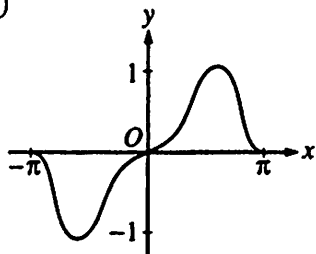
$$\begin{array}{r}
 1\ 4 \\
 4.73 \\
 -4.18 \\
 \hline
 .55 \\
 \hline
 1.4 - 1.1 \\
 .3
 \end{array}$$

$$\begin{array}{r}
 5.5 \\
 \hline
 3
 \end{array}$$

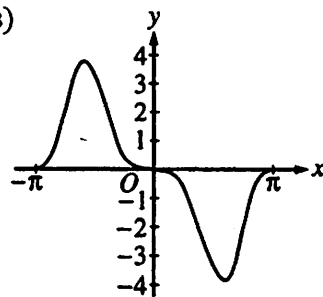
$$1.83$$

21. The graphs of five functions are shown below. Which function has a nonzero average value over the closed interval $[-\pi, \pi]$?

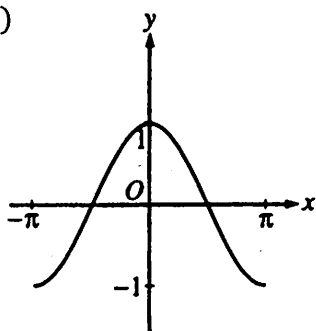
(A)



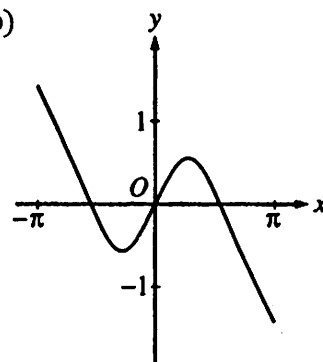
(B)



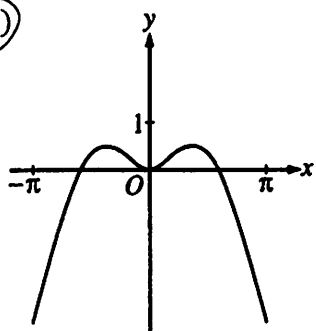
(C)



(D)



(E)



11. What is $\lim_{x \rightarrow \infty} \frac{x^2 - 4}{2 + x - 4x^2}$?

$$\frac{1}{-4}$$

- (A) -2
 (B) $-\frac{1}{4}$
 (C) $\frac{1}{2}$
 (D) 1
 (E) The limit does not exist.

12. The area of the region in the first quadrant between the graph of $y = x\sqrt{4 - x^2}$ and the x -axis is

- (A) $\frac{2}{3}\sqrt{2}$
 (B) $\frac{8}{3}$
 (C) $2\sqrt{2}$
 (D) $2\sqrt{3}$
 (E) $\frac{16}{3}$

$x\sqrt{4-x^2} = 0$
 $x = 2$

$$\int_0^2 x\sqrt{4-x^2} dx$$

$u = 4 - x^2$
 $-\frac{1}{2} du = x dx$

$$-\frac{1}{2} \int_4^0 u^{\frac{1}{2}} du$$

$$\frac{1}{2} \int_0^4 u^{\frac{1}{2}} du = \frac{1}{2} \cdot \frac{2}{3} u^{\frac{3}{2}} \Big|_0^4 = \frac{8}{3}$$

13. Which of the following are antiderivatives of $\frac{\ln^2 x}{x}$?

- I. $\frac{\ln^3 x}{3}$
 II. $\frac{\ln^3 x}{3} + 6$
 III. $\frac{2 \ln x - \ln^2 x}{x^2}$

~~$\frac{\ln^2 x}{x} \cdot \frac{1}{x}$~~

also

~~$\ln x (2 - \ln x)$~~
 $\frac{\ln x (2 - \ln x)}{x^2}$

- (A) I only
 (B) III only
 (C) I and II only
 (D) I and III only
 (E) II and III only

(10)

x	1.1	1.2	1.3	1.4
$f(x)$	4.18	4.38	4.56	4.73

Let f be a function such that $f''(x) < 0$ for all x in the closed interval $[1, 2]$, with selected values shown in the table above. Which of the following must be true about $f'(1.2)$?

- (A) $f'(1.2) < 0$
 (B) $0 < f'(1.2) < 1.6$
 (C) $1.6 < f'(1.2) < 1.8$
 (D) $1.8 < f'(1.2) < 2.0$
 (E) $f'(1.2) > 2.0$

19. Two particles start at the origin and move along the x -axis. For $0 \leq t \leq 10$, their respective position functions are given by $x_1 = \sin t$ and $x_2 = e^{-2t} - 1$. For how many values of t do the particles have the same velocity?

- (A) None
 (B) One
 (C) Two
 (D) Three
 (E) Four

20. If the function g is defined by $g(x) = \int_0^x \sin(t^2) dt$ on the closed interval $-1 \leq x \leq 3$, then g has a local minimum at $x =$

- (A) 0
 (B) 1.084
 (C) $1.772 = \sqrt{\pi}$
 (D) 2.171
 (E) 2.507

Solve $x = \cos x$, $x = 1.5$
 $\rightarrow 739 \rightarrow A$



22. The region in the first quadrant enclosed by the y -axis and the graphs of $y = \cos x$ and $y = x$ is rotated about the x -axis. The volume of the solid generated is

- (A) 0.484
 (B) 0.877
 (C) 1.520
 (D) 1.831
 (E) 3.040

$$\pi \int_0^{\pi/2} (\cos^2 x - x^2) dx$$

$$\int_0^b (x - \cos^2 x) dx$$

Prove working

There is an area beyond $x = \frac{3\pi}{2}$ in 1st quadrant which should be included.

23. Oil is leaking from a tanker at the rate of $R(t) = 2,000e^{-0.2t}$ gallons per hour, where t is measured in hours. How much oil has leaked out of the tanker after 10 hours?

- (A) 54 gallons
 (B) 271 gallons
 (C) 865 gallons
 (D) 8,647 gallons
 (E) 14,778 gallons

#2
 49

$$\int_0^{10} 2000 e^{-0.2t} dt$$

24. If f is continuous for all x , which of the following integrals necessarily have the same value?

I. $\int_a^b f(x) dx$

$$F(b) - F(a)$$

Same.

II. $\int_0^{b-a} f(x+a) dx$

$$F(b-a+a) - F(0+a) \text{ Same}$$

III. $\int_{a+c}^{b+c} f(x+c) dx$

$$F(b+c+c) - F(a+c+c) \text{ No.}$$

#2
 50

- (A) I and II only
 (B) I and III only
 (C) II and III only
 (D) I, II, and III
 (E) No two necessarily have the same value.

#5

(18)
~~add to~~
~~note~~

Part A Sample Multiple-Choice Questions

A calculator may not be used on this part of the examination.

Part A consists of 28 questions. In this section of the examination, as a correction for guessing, one-fourth of the number of questions answered incorrectly will be subtracted from the number of questions answered correctly. Following are the directions for Section I Part A and a representative set of 14 questions.

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

- 85
1. The line perpendicular to the tangent of the curve represented by the equation $y = x^2 + 6x + 4$ at the point $(-2, -4)$ also intersects the curve at $x =$

(A) -6

(B) $-\frac{9}{2}$

(C) $-\frac{7}{2}$

(D) -3

(E) $-\frac{1}{2}$

$$\frac{dy}{dx} = 2x + 6 \Big|_{x=-2} = 2$$

$$\perp \text{ so } m = -\frac{1}{2}$$

$$y + 4 = -\frac{1}{2}(x + 2)$$

$$y + 4 = -\frac{1}{2}x - 1 \quad \text{set } y = x^2 + 6x + 4$$

$$y = -\frac{1}{2}x - 5 = x^2 + 6x + 4$$

$$-x - 10 = 2x^2 + 12x + 8$$

$$2x^2 + 13x + 18 = 0$$

$$(2x + 9)(x + 2)$$
$$x = -\frac{9}{2} \quad x = -2$$

AP Calculus BC

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2. If $y = x + \sin(xy)$, then $\frac{dy}{dx} =$

$$\frac{dy}{dx} = 1 + \cos xy \left[y + x \frac{dy}{dx} \right]$$

$$= 1 + y \cos xy + x \cos xy \frac{dy}{dx}$$

$$\frac{dy}{dx} (1 - x \cos xy) = 1 + y \cos xy$$

$$\frac{dy}{dx} = \frac{1 + y \cos xy}{1 - x \cos xy}$$

- (A) $1 + \cos(xy)$
 (B) $1 + y \cos(xy)$
 (C) $\frac{1}{1 - \cos(xy)}$
 (D) $\frac{1}{1 - x \cos(xy)}$
 (E) $\frac{1 + y \cos(xy)}{1 - x \cos(xy)}$

3. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = \arcsin(xy)$ with the initial condition $f(0) = 2$. What is the approximation for $f(1)$ if Euler's method is used, starting at $x = 0$ with a step size of 0.5?

- (A) 2
 (B) $2 + \frac{\pi}{6}$
 (C) $2 + \frac{\pi}{4}$
 (D) $2 + \frac{\pi}{2}$
 (E) 3

4. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{n3^n}{x^n}$ converges?

- (A) All x except $x = 0$.
 (B) $|x| = 3$
 (C) $-3 \leq x \leq 3$
 (D) $|x| > 3$
 (E) The series diverges for all x .

87

5. If $\frac{d}{dx} f(x) = g(x)$ and if $h(x) = x^2$, then $\frac{d}{dx} f(h(x)) =$

- (A) $g(x^2)$
 (B) $2xg(x)$
 (C) $g'(x)$
 (D) $2xg(x^2)$
 (E) $x^2g(x^2)$

$$f'(h(x)) h'(x)$$

$$g(h(x)) h'(x)$$

$$g(x^2) 2x$$

88

6. If F' is a continuous function for all real x , then $\lim_{h \rightarrow 0} \frac{1}{h} \int_a^{a+h} F'(x) dx$ is

- (A) 0
 (B) $F(0)$
 (C) $F(a)$
 (D) $F'(0)$
 → (E) $F'(a)$

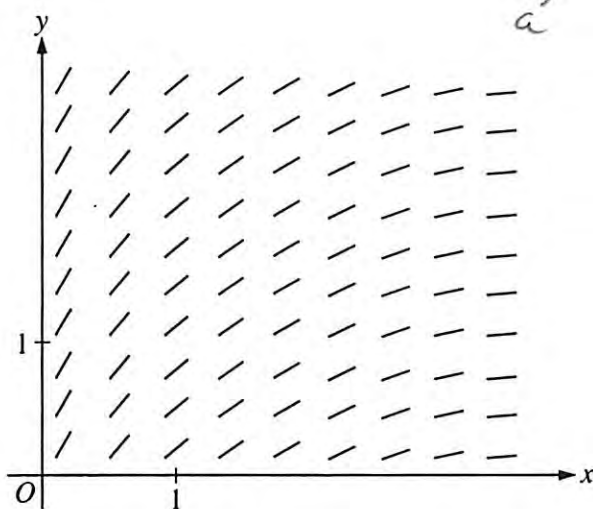
$$\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = f'(a) \text{ def of cont.}$$

$$\int_a^{a+h} f'(x) dx \stackrel{\text{FTC}}{=} f(a+h) - f(a)$$

$$\therefore \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

$$\underbrace{\quad}_{\text{def of der}} f'(a)$$

7.



The slope field for a certain differential equation is shown above. Which of the following could be a specific solution to that differential equation?

- (A) $y = x^2$
 (B) $y = e^x$
 (C) $y = e^{-x}$
 (D) $y = \cos x$
 → (E) $y = \ln x$



89

8. $\int_0^{\infty} e^{-2t} dt$ is

- (A) -1
 (B) $-\frac{1}{2}$
 → (C) $\frac{1}{2}$
 (D) 1
 (E) divergent

$$\lim_{b \rightarrow \infty} \left. -\frac{1}{2e^{2t}} \right|_0^b$$

$$\lim_{b \rightarrow \infty} \left(-\frac{1}{2e^{2b}} + \frac{1}{2} \right) = -0 + \frac{1}{2} = \frac{1}{2}$$

8. Which of the following series converge to 2?

I. $\sum_{n=1}^{\infty} \frac{2n}{n+3}$

II. $\sum_{n=1}^{\infty} \frac{-8}{(-3)^n}$

III. $\sum_{n=0}^{\infty} \frac{1}{2^n}$

- (A) I only
(B) II only
(C) III only
(D) I and III only
(E) II and III only

10. What is the area of the closed region bounded by the curve $y = e^{2x}$ and the lines $x = 1$ and $y = 1$?

90

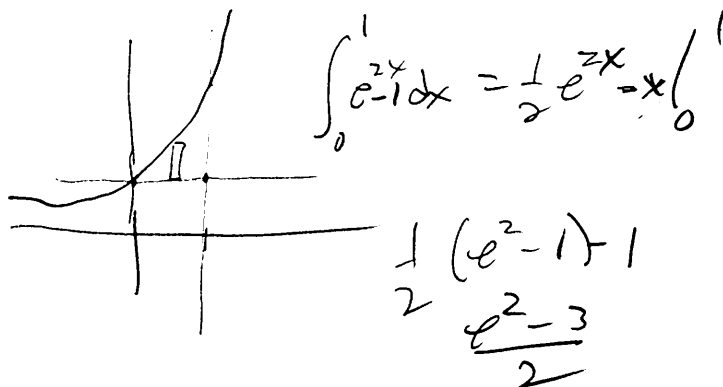
(A) $\frac{2 - e^2}{2}$

→ (B) $\frac{e^2 - 3}{2}$

(C) $\frac{3 - e^2}{2}$

(D) $\frac{e^2 - 2}{2}$

(E) $\frac{e^2 - 1}{2}$



91 11. Which of the following integrals gives the length of the graph of $y = \sqrt{x}$ between $x = a$ and $x = b$, where $0 < a < b$?

(A) $\int_a^b \sqrt{x^2 + x} dx$

(B) $\int_a^b \sqrt{x + \sqrt{x}} dx$

(C) $\int_a^b \sqrt{x + \frac{1}{2\sqrt{x}}} dx$

(D) $\int_a^b \sqrt{1 + \frac{1}{2\sqrt{x}}} dx$

→ (E) $\int_a^b \sqrt{1 + \frac{1}{4x}} dx$

Handwritten derivative: $y' = \frac{1}{2\sqrt{x}}$

Handwritten formula for arc length: $\sqrt{1 + (y')^2}$

Handwritten calculation: $(y')^2 = \frac{1}{4x}$

Handwritten result: $\sqrt{1 + \frac{1}{4x}}$

- 92 ✓ 12. A point (x, y) is moving along a curve $y = f(x)$. At the instant when the slope of the curve is $-\frac{1}{3}$, the x -coordinate of the point is increasing at the rate of 5 units per second. The rate of change, in units per second, of the y -coordinate of the point is

- (A) $-\frac{5}{3}$
 (B) $-\frac{1}{3}$
 (C) $\frac{1}{3}$
 (D) $\frac{3}{5}$
 (E) $\frac{5}{3}$

$$\begin{aligned} \frac{dy}{dx} &= -\frac{1}{3} \\ \frac{dy}{dt} &= 5 \\ \frac{dy}{dx} &= \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \\ \frac{dy}{dx} \cdot \frac{dx}{dt} &= \frac{dy}{dt} \\ \frac{dy}{dx} \cdot 5 &= 5 \\ \frac{dy}{dx} &= -\frac{1}{3} \\ 5 \cdot \left(-\frac{1}{3}\right) &= -\frac{5}{3} \end{aligned}$$

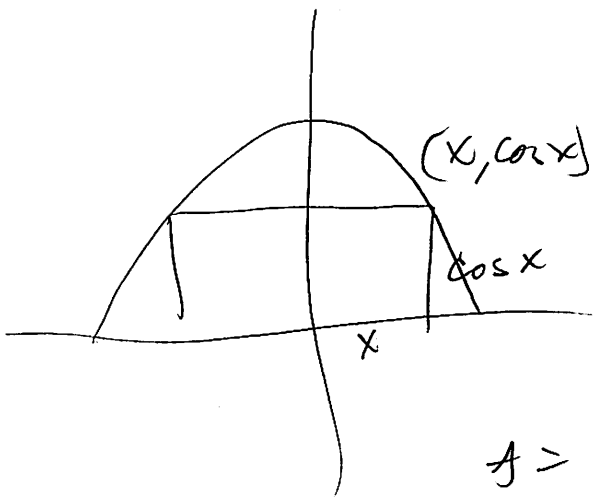
13. The third-degree Taylor polynomial about $x = 0$ of $\ln(1 - x)$ is

- (A) $-x - \frac{x^2}{2} - \frac{x^3}{3}$
 (B) $1 - x + \frac{x^2}{2}$
 (C) $x - \frac{x^2}{2} + \frac{x^3}{3}$
 (D) $-1 + x - \frac{x^2}{2}$
 (E) $-x + \frac{x^2}{2} - \frac{x^3}{3}$

- 93 14. If $\frac{dy}{dx} = y \sec^2 x$ and $y = 5$ when $x = 0$, then $y =$

- (A) $e^{\tan x} + 4$
 (B) $e^{\tan x} + 5$
 (C) $5e^{\tan x}$
 (D) $\tan x + 5$
 (E) $\tan x + 5e^x$

$$\begin{aligned} \frac{dy}{y} &= \sec^2 x \, dx \\ \ln y &= \tan x + C \\ y &= A e^{\tan x} \\ y &= 5 e^{\tan x} \end{aligned}$$



$$A = 2x \cos x$$

$$A' = \cos x - 2x \sin x \stackrel{\text{Set}}{=} 0$$

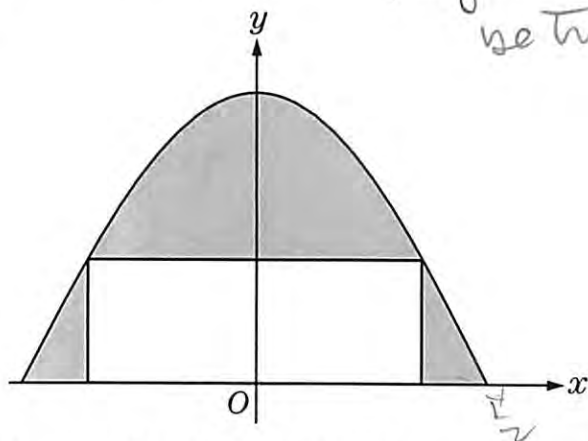
$$\cos x = 2x \sin x$$

$$\cot x = 2x$$

16. If f is differentiable at $x = a$, which of the following could be false?

- 94
- (A) f is continuous at $x = a$. *must be cont to have a der.*
 - (B) $\lim_{x \rightarrow a} f(x)$ exists. *Part of Def of Cont.*
 - (C) $\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$ exists. *Def of der*
 - (D) $f'(a)$ is defined. *Part of Def of Cont.*
 - (E) $f''(a)$ is defined. *may or may not be true.*

17.



A rectangle with one side on the x -axis has its upper vertices on the graph of $y = \cos x$, as shown in the figure above. What is the minimum area of the shaded region?

- min area when rect is largest.
- (A) 0.799
 - (B) 0.878
 - (C) 1.140
 - (D) 1.439
 - (E) 2.000

$$A_{\text{rect}} = 2x \cos x$$

$$\int_{-\pi/2}^{\pi/2} \cos x \, dx - B$$

$$A' = 2 \cos x - 2x \sin x \stackrel{u,v}{=} 2 \cos x - 2x \sin x$$

$$x = \cot x$$

$$x = .860$$

$$A'' = -2 \sin x$$

$$-2 \sin x - 2x \cos x$$

$$-4 \sin x - 2x \cos x$$

$$A''(A) = -4.154$$

AP Calculus BC

$\therefore$ Max $f''(x)$

18.

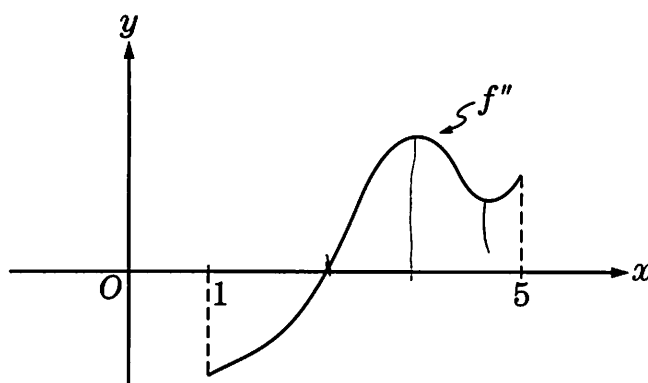
Time (sec)	0	10	25	37	46	60
Rate (gal/sec)	500	400	350	280	200	180

The table above gives the values for the rate (in gal/sec) at which water flowed into a lake, with readings taken at specific times. A right Riemann sum, with the five subintervals indicated by the data in the table, is used to estimate the total amount of water that flowed into the lake during the time period $0 \leq t \leq 60$. What is this estimate?

- (A) 1,910 gal
(B) 14,100 gal
(C) 16,930 gal
(D) 18,725 gal
(E) 20,520 gal

$$10 \cdot 400 + 15 \cdot 350 + 12 \cdot 280 + 9 \cdot 200 + 14 \cdot 180$$

19.



Let f be a function whose domain is the open interval $(1, 5)$. The figure above shows the graph of f'' . Which of the following describes the relative extrema of f' and the points of inflection of the graph of f' ?

- (A) 1 relative maximum, 1 relative minimum, and no point of inflection
(B) 1 relative maximum, 2 relative minima, and no point of inflection
(C) 1 relative maximum, 1 relative minimum, and 1 point of inflection
(D) 1 relative maximum and 2 points of inflection
(E) 1 relative minimum and 2 points of inflection

no other extrema values for f'
when slope of $f'' = 0$ AND f'' changes from $+$ to $-$ or $-$ to $+$ PI in f'
2 PI
and
1 min



97

20. A particle moves along the x -axis so that at any time $t \geq 0$ its velocity is given by $v(t) = \ln(t+1) - 2t + 1$. The total distance traveled by the particle from $t = 0$ to $t = 2$ is

- (A) 0.667
(B) 0.704
(C) 1.540
(D) 2.667
(E) 2.901

- .704 if
also left
off

$$\int_0^2 |\ln(t+1) - 2t + 1| dt$$

99

21. If the function f is defined by $f(x) = \sqrt{x^3 + 2}$ and g is an antiderivative of f such that $g(3) = 5$, then $g(1) =$

- (A) -3.268
(B) -1.585
(C) 1.732
(D) 6.585
(E) 11.585

$g = \int \sqrt{x^3 + 2} dx$
 $\int_1^3 f(x) dx = f(3) - f(1) = 6.585$
 $g(1) = f(1) = -f(3) + 6.585 = -1.585$

22. Let g be the function given by $g(x) = \int_1^x 100(t^2 - 3t + 2)e^{-t^2} dt$. Which

- I. g is increasing on $(1, 2)$. No
II. g is increasing on $(2, 3)$. Yes
III. $g(3) > 0$. No

100

- (A) I only
(B) II only
(C) III only
(D) II and III only
(E) I, II, and III

Yes.

$g'(x) = 100(x^2 - 3x + 2)e^{-x^2}$
 $(x-2)(x-1)$
 Sign chart for g' :
 + - +
 1 2 3

Let $g(x) = 5 + \int_3^x \sqrt{t^3 + 2} dt$ because $g(3) = 5 + \int_3^3 \sqrt{t^3 + 2} dt = 5$
 $g(1) = 5 + \int_3^1 \sqrt{t^3 + 2} dt$
 $5 - 6.584 = -1.585$

$$f(t) = 100 + 20 \sin \frac{\pi t}{2} + 10 \cos \frac{\pi t}{6}$$

$$f'(t) = 10\pi \cos \frac{\pi t}{2} - \frac{5\pi}{3} \sin \frac{\pi t}{6}$$

$$f''(t) = -5\pi^2 \sin \frac{\pi t}{2} - \frac{5\pi^2}{18} \cos \frac{\pi t}{6}$$

23. The area of one loop of the graph of the polar equation $r = 2 \sin(3\theta)$ is given by which of the following expressions?

(A) $4 \int_0^{\frac{\pi}{3}} \sin^2(3\theta) d\theta$

(B) $2 \int_0^{\frac{\pi}{3}} \sin(3\theta) d\theta$

(C) $2 \int_0^{\frac{\pi}{3}} \sin^2(3\theta) d\theta$ ←

(D) $2 \int_0^{\frac{2\pi}{3}} \sin^2(3\theta) d\theta$

(E) $2 \int_0^{\frac{2\pi}{3}} \sin(3\theta) d\theta$

24. Let g be the function given by $g(t) = 100 + 20 \sin\left(\frac{\pi t}{2}\right) + 10 \cos\left(\frac{\pi t}{6}\right)$.

For $0 \leq t \leq 8$, g is decreasing most rapidly when $t =$

(A) 0.949

(B) 2.017

(C) 3.106

(D) 5.965

(E) 8.000

Answers to Calculus BC Multiple-Choice Questions

Part A

1. B

2. E

3. C

4. D

5. D

6. E

7. E

8. C

9. E

10. B

11. E

12. A

13. A

14. C

Part B

15. A

16. E

17.* B

18. C

19. E

20.* C

21.* B

22.* B

23. C

24.* B

* Indicates a graphing calculator-active question.

A P CALCULUS AB

*underlined = talk about
circled = have students
put on boards*

Mr. Mumaugh

MULTIPLE-GUESS #1

- | | | | | | | | | |
|--------------|-------|-------|-------------|-------|-------------|--------------|-------|-------|
| 1) C | 2) A | 3) D | 4) <u>C</u> | 5) E | 6) <u>D</u> | 7) D | 8) A | 9) C |
| 10) E | 11) B | 12) A | 13) B | 14) D | 15) A | 16) E | 17) C | 18) D |
| 19) B | 20) C | 21) A | 22) E | 23) D | 24) B | 25) B | 26) E | 27) C |
| 28) A | 29) D | 30) A | 31) C | 32) B | 33) E | 34) <u>B</u> | 35) D | 36) X |
| 37) <u>A</u> | 38) X | 39) B | 40) E | 41) D | 42) B | 43) C | 44) D | 45) E |

*10 board
pb/ins*

MULTIPLE-GUESS #2

- | | | | | | | | | |
|-------|--------------|--------------|-------|-------|-------|-------|-------|-------|
| 1) C | 2) A | 3) <u>B</u> | 4) C | 5) E | 6) C | 7) A | 8) D | 9) E |
| 10) C | 11) C | 12) B | 13) D | 14) E | 15) C | 16) B | 17) B | 18) A |
| 19) D | 20) <u>E</u> | 21) D | 22) A | 23) E | 24) E | 25) B | 26) D | 27) C |
| 28) C | 29) E | 30) <u>A</u> | 31) B | 32) C | 33) B | 34) E | 35) X | 36) A |
| 37) D | 38) B | 39) C | 40) D | 41) X | 42) X | 43) E | 44) A | 45) B |

*14
board
pb/ins*

MULTIPLE-GUESS #3

- | | | | | | | | | |
|--------------|-------|-------|-------|-------|--------------|-------|------------------|-------|
| 1) E | 2) A | 3) C | 4) A | 5) D | 6) E | 7) D | 8) D | 9) E |
| 10) C | 11) B | 12) C | 13) B | 14) E | 15) C | 16) D | 17) ~~X~~ | 18) A |
| 19) B | 20) A | 21) C | 22) E | 23) A | 24) B | 25) D | 26) B | 27) C |
| 28) <u>E</u> | 29) D | 30) A | 31) C | 32) B | 33) A | 34) D | 35) C | 36) C |
| 37) E | 38) B | 39) C | 40) X | 41) D | 42) <u>A</u> | 43) E | 44) D | 45) B |

*12 board
pb/ins*

MULTIPLE-GUESS #4

- | | | | | | | | | |
|-------|-------|--------------|-------|-------|-------|-------|-------|-------|
| 1) A | 2) C | 3) <u>E</u> | 4) D | 5) B | 6) E | 7) D | 8) A | 9) C |
| 10) D | 11) A | 12) <u>B</u> | 13) B | 14) C | 15) D | 16) E | 17) X | 18) C |
| 19) X | 20) D | 21) C | 22) C | 23) D | 24) A | 25) B | 26) E | 27) C |
| 28) D | 29) A | 30) E | 31) C | 32) B | 33) D | 34) A | 35) X | 36) X |
| 37) D | 38) X | 39) D | 40) A | 41) X | 42) X | 43) E | 44) D | 45) C |

*10 board
pb/ins*

Test #5 Put on Boards

*18, 33, 35, 36, 39, 42, 44, 51
58, 65, 78, 84*

*Test #5
Put on Boards*

Practice Examination: 1

1. $\lim_{x \rightarrow \infty} \frac{3x^2 - 4}{x^2 - 7x - x^2}$ is

- (A) 3 (B) 1 (C) -3 (D) ∞ (E) 0.

2. The value of k for which the line $3x + ky - 5 = 0$ is perpendicular to the line $x - 2y + 7 = 0$ is

- (A) $\frac{3}{2}$ (B) $-\frac{3}{2}$ (C) 6 (D) -6 (E) $\frac{2}{3}$.

3. $\lim_{h \rightarrow 0} \frac{\cos(\frac{\pi}{2} + h)}{h}$ is

- (A) 1 (B) nonexistent (C) 0 (D) -1 (E) none of these.

4. The distance between the point $(1, -2)$ and the line $4x - 3y - 5 = 0$ is

- (A) $\sqrt{10}$ (B) 2 (C) 1 (D) $\frac{16}{5}$ (E) none of these.

5. The equation of the parabola with vertex at $(2, 1)$ and focus at $(2, 0)$ is

- (A) $x^2 - 4x - 4y + 8 = 0$ (B) $4y^2 - 8y + x + 2 = 0$
 (C) $y^2 - 4x - 2y + 9 = 0$ (D) $x^2 - 2x + 4y - 7 = 0$
 (E) $x^2 - 4x + 4y = 0$.

6. The graph of $(x + 1)^2 - (y - 3)^2 = 0$ consists of

- (A) a hyperbola (B) a point (C) an ellipse
 (D) two intersecting lines (E) two parallel lines.

These are asymptotes of through $(-1, 3)$

Case I: $y - 3 = x + 1 \Rightarrow y = x + 4$

Case II: $y - 3 = -(x + 1) \Rightarrow y = -x + 2$

$f' = 4x^3 - 12x^2 = 0$
 $4x(x-3) = 0$
 $f'' = 12x^2 - 24x = 0$
 $12(x^2 - 2x) = 0$
 $12x(x-2) = 0$
 $f(0) = 6$ (outside domain)
 $f(2) = 6$ (max value)
 Practice Examination: 1 255

7. The maximum value of the function $f(x) = x^4 - 4x^3 + 6$ on the closed interval $[1, 4]$ is

(A) 1 (B) 0 (C) 3 (D) 6 (E) none of these.

Global max at $x = -1$ value 11. But this is outside domain.

8. The equation of the cubic with a relative maximum at $(0, 0)$ and a point of inflection at $(1, -2)$ is

$y = ax^3 + bx^2 + cx + d$
 $y' = 3ax^2 + 2bx + c$
 $y'' = 6ax + 2b$
 $y(0) = d = 0$
 $3a + b = 0$
 $-a - b = 2$
 $b = -3$
 $a = 1$
 $y = x^3 - 3x^2$

(A) $y = x^3 - 3x^2$ (B) $y = -x^3 + 3x^2$

(C) $y = 4x^3 - 6x^2$ (D) $y = x^3 - 3x$

(E) $y = 3x - x^3$

9. If $f(x)$ is continuous at the point where $x = a$, which of the following statements may be false?

(A) $\lim_{x \rightarrow a} f(x)$ exists

(B) $\lim_{x \rightarrow a} f(x) = f(a)$

(C) $f'(a)$ exists

(D) $f(a)$ is defined

(E) $\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x)$

10. Which of the following functions is not everywhere continuous?

(A) $y = |x|$ (B) $y = \frac{x}{x^2 + 1}$ (C) $y = \sqrt{x^2 + 8}$

(D) $y = x^{2/3}$ (E) $y = \frac{4}{(x+1)^2}$

11. The equation of the tangent to the curve of $y = x^2 - 4x$ at the point where the curve crosses the y -axis is

(A) $y = 8x - 4$ (B) $y = -4x$ (C) $y = -4$

(D) $y = 4x$ (E) $y = 4x - 8$

12. If y is a differentiable function of x , then the slope of the curve of $xy^2 - 2y + 4y^3 = 6$ at the point where $y = 1$ is

(A) $-\frac{1}{18}$ (B) $-\frac{1}{26}$ (C) $\frac{5}{18}$ (D) $-\frac{11}{18}$ (E) 2

$2xy \frac{dy}{dx} + y^2 - 2 + 12y^2 = 0$
 $(2xy - 2 + 12y^2) \frac{dy}{dx} = -y^2 \Rightarrow (8 - 2 + 12) \frac{dy}{dx} = -1$
 $\frac{dy}{dx} = -\frac{1}{18}$

13. If $x = 2t - 1$ and $y = 3 - 4t^2$ then $\frac{dy}{dx}$ is

(A) $4t$ (B) $-4t$ (C) $-\frac{1}{4t}$ (D) $2(x+1)$
 (E) $-4(x+1)$

- *14. A curve is given parametrically by $x = e^t$ and $y = 2e^{-t}$. The equation of the tangent to the curve at $t = 0$ is

(A) $2x + y = 0$ (B) $x + 2y = 5$ (C) $y = -2x + 5$
 (D) $2x + y = 4$ (E) $y = 2x$

15. If, for all x , $f'(x) = (x-2)^4(x-1)^3$, it follows that the function f has

(A) a relative minimum at $x = 1$
 (B) a relative maximum at $x = 1$
 (C) both a relative minimum at $x = 1$ and a relative maximum at $x = 2$
 (D) neither a relative maximum nor a relative minimum
 (E) relative minima at $x = 1$ and at $x = 2$

16. If a particle's motion along a straight line is given by

$$s = t^3 - 6t^2 + 9t + 2, \text{ then } s \text{ is increasing}$$

(A) when $1 < t < 3$ (B) when $-1 < t < 3$
 (C) for all t (D) when $|t| > 3$
 (E) when $t < 1$ or $t > 3$

17. Which of the following statements is not true of the graph of $y^2 = x^3 - x$?

(A) It is symmetric to the x -axis.
 (B) It intersects the x -axis at 0, 1, and -1.
 (C) It exists only for $|x| \leq 1$.
 (D) It has no horizontal asymptotes.
 (E) It has no vertical asymptotes.

18. The area in the first quadrant bounded by the curve $y = x^2$ and the line $y - x - 2 = 0$ is equal to

(A) $\frac{3}{2}$ (B) $\frac{2}{3}$ (C) $\frac{7}{6}$ (D) $\frac{10}{3}$ (E) $\frac{9}{2}$

- *19. If differentials are used for the evaluation, then $\sqrt[4]{15}$ is approximately equal to

(A) 2.97 (B) 1.97 (C) 2.03 (D) 1.99 (E) 1.94

20. The curve of the equation $(x^2 - 1)y = x^2 - 4$ has

(A) one horizontal and one vertical asymptote
(B) two vertical but no horizontal asymptotes
(C) one horizontal and two vertical asymptotes
(D) two horizontal and two vertical asymptotes
(E) neither a horizontal nor a vertical asymptote.

- *21. If $\frac{dy}{dx} = \cos x \cos^2 y$ and $y = \frac{\pi}{4}$ when $x = 0$, then

(A) $\tan y = \sin x + 1$ (B) $\tan y = -\sin x + 1$
(C) $\sec^2 y = \sin x + 2$ (D) $\tan y = \frac{1}{2}(\cos^2 x + 1)$
(E) $\tan y = \sin x - \frac{\sqrt{2}}{2}$

22. If $f(x) = \cos x \sin 3x$, then $f'(\frac{\pi}{6})$ is equal to

(A) $\frac{1}{2}$ (B) $-\frac{\sqrt{3}}{2}$ (C) 0 (D) 1 (E) $-\frac{1}{2}$

- *23. The region in the first quadrant bounded by the x -axis, the y -axis, and the curve of $y = e^{-x}$ is rotated about the x -axis. The volume of the solid obtained is equal to

(A) π (B) 2π (C) $\frac{1}{2}$ (D) $\frac{\pi}{2}$ (E) none of these.

24. If $y = x^2 \ln x$, ($x > 0$), then y'' is equal to

(A) $3 + \ln x$ (B) $3 + 2 \ln x$ (C) $3 \ln x$
(D) $3 + 3 \ln x$ (E) $2 + x + \ln x$

25. $\int_0^1 \frac{x \, dx}{x^2 + 1}$ is equal to
- (A) $\frac{\pi}{4}$ (B) $\ln \sqrt{2}$ (C) $\frac{1}{2}(\ln 2 - 1)$ (D) $\frac{3}{2}$ (E) $\ln 2$.
26. $\int_0^{\pi/2} \sin^2 x \, dx$ is equal to
- (A) $\frac{1}{3}$ (B) $\frac{\pi}{4} - \frac{1}{4}$ (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{2} - \frac{1}{3}$ (E) $\frac{\pi}{4}$.

27. The acceleration of a particle moving along a straight line is given

by $a = 6t$. If, when $t = 0$, its velocity, v , is 1 and its distance, s , is 3, then at any time t

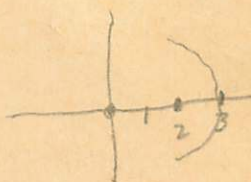
- (A) $s = t^3 + 3$ (B) $s = t^3 + 3t + 1$ (C) $s = t^3 + t + 3$
 (D) $s = \frac{t^3}{3} + t + 3$ (E) $s = \frac{t^3}{3} + \frac{t^2}{2} + 3$.

28. If $y = f(x^2)$ and $f'(x) = \sqrt{5x - 1}$, then $\frac{dy}{dx}$ is equal to

- (A) $2x\sqrt{5x^2 - 1}$ (B) $\sqrt{5x - 1}$ (C) $2x\sqrt{5x - 1}$
 (D) $\frac{\sqrt{5x - 1}}{2x}$ (E) none of these.

29. The equation of the ellipse with center at the origin and with a focus and a vertex respectively at $(2, 0)$ and $(3, 0)$ is

- (A) $13x^2 + 9y^2 = 117$ (B) $9x^2 + 5y^2 = 45$
 (C) $5x^2 - 9y^2 = 45$ (D) $5x^2 + 9y^2 = 45$
 (E) $5x^2 + 4y^2 = 20$.



30. $\int_0^1 xe^x \, dx$ equals
- (A) 1 (B) -1 (C) $2 - e$ (D) $\frac{e^2}{2} - e$ (E) $e - 1$.

31. A 26-foot ladder leans against a building so that its foot moves away from the building at the rate of 3 feet per second. When the foot of the ladder is 10 feet from the building, then the top is moving down at the rate of r ft/sec, where r is

$$\frac{dr}{dt} = \frac{-30}{24} = -\frac{5}{4}$$

$$x^2 + y^2 = 26^2$$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

- (A) $\frac{46}{3}$ (B) $\frac{3}{4}$ (C) $\frac{5}{4}$ (D) $\frac{5}{2}$ (E) $\frac{4}{5}$.

- *32. If the arc of the curve $y^2 = x$ between $(0,0)$ and $(2,\sqrt{2})$ is rotated about the x -axis, then the area of the surface generated is equal to

- (A) $\frac{104\pi}{3}$ (B) $\frac{13\pi}{3}$ (C) $\frac{\pi}{6} (17\sqrt{17} - 1)$

- (D) 5π (E) none of these.

33. If $\frac{dx}{dt} = kx$ and if $x = 2$ when $t = 0$ and $x = 6$ when $t = 1$, then

k equals

- (A) $\ln 4$ (B) 8 (C) e^3 (D) 3 (E) none of these.

34. The locus of the polar equation $r = 2 \sec \theta$ is

- (A) a circle (B) a vertical line
(C) a horizontal line (D) a parabola
(E) an oblique line through the pole.

35. If $f(x) = \sqrt{1 - \frac{2}{x}}$ and $g(x) = \frac{1}{x}$, ($x \neq 0$), then the derivative of

$f(g(x))$ is equal to

- (A) 0 (B) $\frac{1}{x^2 \sqrt{1 - \frac{2}{x}}}$ (C) does not exist

- (D) $-\frac{1}{\sqrt{1 - 2x}}$ (E) $\frac{-2}{(x^2 - 2)^{3/2}}$.

- *36. A hemispherical tank of radius 4 ft is filled with a liquid weighing w lb/ft³. The work done in pumping all the liquid just to the top of the tank, expressed in foot-pounds, is

- (A) $128\pi w$ (B) $\frac{256}{3}\pi w$ (C) $\frac{320}{3}\pi w$ (D) $96\pi w$ (E) $64\pi w$.

continuous on closed interval

37. Which one of the following functions does not satisfy the hypotheses of Rolle's Theorem?

(A) $f(x) = \frac{x^2 - 9x}{x - 2}$

(B) $f(x) = x^2 - 3x$

(C) $f(x) = \frac{x^2 - 1}{x^2 + 1}$

(D) $f(x) = (x - 1)(e^x - 1)$

(E) $f(x) = \frac{x^2 - x}{x + 1}$

interval defined when $f'(x) = 0$

$x(x-9) = 0$ discontinuity occurs within interval at $f(a) = f(b)$

$x(x-1) = 0$ discontinuity occurs at $x = -1$ which is not in interval.

*38. $\lim_{n \rightarrow \infty} \frac{1}{n} \left[\frac{1}{n^2} + \left(\frac{2}{n}\right)^2 + \left(\frac{3}{n}\right)^2 + \dots + \left(\frac{n}{n}\right)^2 \right]$ is

(A) $\ln 2$

(B) $\frac{1}{4}$

(C) $\frac{1}{3}$

(D) 0

(E) ∞

39. The first quadrant area under the curve $y = \frac{1}{\sqrt{1-x^2}}$ and bounded

at the right by $x = 1$ is

(A) ∞

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{4}$

(D) 2

(E) none of these.

40. The curve of $y = \frac{1-x}{x-3}$ is concave up when

(A) $x > 3$

(B) $1 < x < 3$

(C) $x > 1$

(D) $x < 1$

(E) $x < 3$

$\frac{-(x-3) - (1-x)}{(x-3)^2} = \frac{-x+3-1+x}{(x-3)^2} = \frac{2}{(x-3)^2}$

$y'' = \frac{-2 \cdot 2}{(x-3)^3} = \frac{-4}{(x-3)^3}$

41. The area of the largest isosceles triangle that can be drawn with one vertex at the origin and with the others on a line parallel to the

x -axis and on the curve $y = 27 - x^2$ is

(A) 108

(B) 27

(C) $12\sqrt{3}$

(D) 54

(E) $24\sqrt{3}$

$A = 27x - x^3$
 $A' = 27 - 3x^2 = 0 \Rightarrow x = 3$

42. The average (mean) value of $\tan x$ on the interval from $x = 0$ to

$x = \frac{\pi}{3}$ is

(A) $\ln 2$

(B) $\frac{3}{\pi} \ln 2$

(C) $\ln \frac{1}{2}$

(D) $\frac{9}{\pi}$

(E) $\frac{\sqrt{3}}{2}$

$\frac{3}{\pi} \int_0^{\pi/3} \tan x \, dx = \frac{3}{\pi} \int_{\frac{1}{2}}^1 \frac{du}{u} = \frac{3}{\pi} \ln u \Big|_{\frac{1}{2}}^1 = 0 - \frac{3}{\pi} \ln \frac{1}{2} = \frac{3}{\pi} \ln 2$

area $\frac{1}{2}xy$



$u = \cos x$
 $du = -\sin x \, dx$
 $x = 0 \Rightarrow u = 1$
 $x = \frac{\pi}{3} \Rightarrow u = \frac{1}{2}$

- *43. A particle moves along the curve given parametrically by $x = \tan t$ and $y = \sec t$. At the instant when $t = \frac{\pi}{6}$, its speed equals

(A) $\sqrt{2}$ (B) $2\sqrt{7}$ (C) $\frac{2\sqrt{5}}{3}$ (D) $\frac{2\sqrt{13}}{3}$ (E) none of these.

44. If $y = \sin(x^2 - 1)$ and $x = \sqrt{u^2 + 1}$, then $\frac{dy}{du}$ equals

(A) $\frac{u \cos u^2}{\sqrt{1 + u^2}}$ (B) $\cos u^2$ (C) $\frac{\cos u^2}{2\sqrt{1 + u^2}}$ (D) $2u \cos u^2$ (E) $\frac{u \cos(x^2 - 1)}{\sqrt{1 + u^2}}$

45. If $f(x) = x^n$, n a positive integer, the first derivative of $f(x)$ which is identically zero is $n! x^{n-n}$ one more to zero.

(A) the n th (B) the $(n - 1)$ st (C) the $(n + 2)$ nd
(D) the first (E) the $(n + 1)$ st.

(43)

$$\frac{dx}{dt} = \sec^2 t$$

$$\frac{dy}{dt} = \sec t \tan t$$

$$\frac{dy}{dx} = \frac{\sec t \tan t}{\sec^2 t} = \frac{\tan t}{\sec t} = \frac{\sin t}{\cos t} \cdot \frac{\cos t}{1} = \frac{\sin t}{1} = \frac{1}{2}$$

(43)

$$\frac{1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\frac{4}{3} \left(\frac{4}{3} + \frac{1}{3} \right) = \frac{20}{9} = \frac{2\sqrt{5}}{3}$$

$$x \rightarrow 0 \quad \theta \rightarrow 0$$

$$\frac{\sin \theta}{\theta}$$

$$\lim_{\theta \rightarrow 0}$$

$$-1 \leq \sin \theta \leq 1$$

$$-\frac{1}{\theta} \leq \sin \theta \leq \frac{1}{\theta}$$

$$\text{Now take } \lim_{\theta \rightarrow 0}$$

$$0 \leq \lim_{\theta \rightarrow 0}$$

$$\frac{\sin \theta}{\theta} \leq 0$$

$$\lim_{\theta \rightarrow 0} = 0$$

Entered Ph/m 38 on New version
#1 missing highlighted
event 100

But $\sin \theta$ oscillates
so it crosses from $+$ to $-$
ie it crosses in on 2nd
but does NOT approach
a particular number.

Practice Examination: 2

1. $\lim_{x \rightarrow \infty} \frac{20x^2 - 13x + 5}{5 - 4x^3}$ is $\frac{20}{-4} = 0$

(A) -5 (B) ∞ (C) 0 (D) 5 (E) 1

2. $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos x}{x - \frac{\pi}{2}}$ is

use l'Hopital's rule

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{-\sin x}{1} = -1$$

or def of der of
 $f(x) = \cos x$ evaluated
at $x = \frac{\pi}{2}$

(A) -1 (B) 1 (C) 0 (D) ∞ (E) none of these.

3. $\lim_{x \rightarrow 0} x \sin \frac{1}{x}$ is

$$\lim_{x \rightarrow 0} \frac{\sin \frac{1}{x}}{\frac{1}{x}} = 0$$

$$\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$$

(A) 1 (B) 0 (C) ∞ (D) -1 (E) none of these.

4. $\lim_{h \rightarrow 0} \frac{\ln(2+h) - \ln 2}{h}$ is

def of $f'(x)$ where $f(x) = \ln x$

$$f'(x) = \frac{1}{x} \bigg|_{x=2} = \frac{1}{2}$$

(A) 0 (B) $\ln 2$ (C) $\frac{1}{2}$ (D) $\frac{1}{\ln 2}$ (E) ∞

$$y'' = -2e^{-x^2} \quad \Big|_{x=1} = -2e^{-1} = -\frac{2}{e}$$

#6

$$y = e^{-x^2}$$

$$y' = -2xe^{-x^2}$$

$$y'' = -2e^{-x^2} - 2x(-2xe^{-x^2})$$

$$= -2e^{-x^2} + 4x^2e^{-x^2} \quad \Big|_{x=1}$$

$$= -\frac{2}{e} + \frac{4}{e} = \frac{2}{e}$$

5. If $y = \frac{x-3}{2-5x}$ then $\frac{dy}{dx}$ equals

(A) $\frac{17-10x}{(2-5x)^2}$ (B) $\frac{13}{(2-5x)^2}$ (C) $\frac{x-3}{(2-5x)^2}$
 (D) $\frac{17}{(2-5x)^2}$ (E) $\frac{-13}{(2-5x)^2}$

6. If $y = e^{-x^2}$ then y'' equals

(A) 2 (B) -2 (C) $\frac{2}{e}$ (D) 0 (E) -4

7. If $f(x) = x \cos \frac{1}{x}$, then $f'(\frac{2}{\pi})$ equals

(A) $\frac{\pi}{2}$ (B) $-\frac{2}{\pi}$ (C) -1 (D) $-\frac{\pi}{2}$ (E) $1 + \frac{\pi}{2}$

8. If $xy^2 - 3x + 4y - 2 = 0$ and y is a differentiable function of x , then $\frac{dy}{dx}$ equals

(A) $-\frac{1+y^2}{2xy}$ (B) $\frac{3}{2y+4}$ (C) $\frac{3}{2xy+4}$
 (D) $\frac{3-y^2}{2xy+4}$ (E) $\frac{5-y^2}{2xy+4}$

9. If $x = \sqrt{1-t^2}$ and $y = \sin^{-1} t$ then $\frac{dy}{dx}$ equals

(A) $-\frac{\sqrt{1-t^2}}{t}$ (B) -t (C) $\frac{t}{1-t^2}$
 (D) 2 (E) $-\frac{1}{t}$

10. If y is a differentiable function of x , then the derivative of $\sin^2(x+y)$ with respect to x is

(A) $2[\sin(x+y)] \frac{dy}{dx}$ (B) $[\cos^2(x+y)](1 + \frac{dy}{dx})$
 (C) $[\sin 2(x+y)](1 + \frac{dy}{dx})$ (D) $\cos^2(x+y)$
 (E) $2 \sin(x+y)$

$$\frac{2-5x+5(x-3)}{(2-5x)^2} = \frac{-13}{(2-5x)^2}$$

$$y' = -2xe^{-x^2} \quad y'' = -2e^{-x^2} + 4x^2e^{-x^2}$$

$$f' = \cos \frac{1}{x} + x(-\sin \frac{1}{x})(-\frac{1}{x^2}) = \cos \frac{1}{x} + \frac{1}{x} \sin \frac{1}{x}$$

$$2xy \frac{dy}{dx} + y^2 - 3 + 4 \frac{dy}{dx} = 0$$

$$(2xy+4) \frac{dy}{dx} = 3-y^2$$

$$\frac{dx}{dt} = \frac{-2t}{2\sqrt{1-t^2}}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-t^2}}$$

$$2 \sin(x+y) \cos(x+y) (1 + \frac{dy}{dx})$$

- *43. A particle moves along the curve given parametrically by $x = \tan t$ and $y = \sec t$. At the instant when $t = \frac{\pi}{6}$, its speed equals
- (A) $\sqrt{2}$ (B) $2\sqrt{7}$ (C) $\frac{2\sqrt{5}}{3}$ (D) $\frac{2\sqrt{13}}{3}$ (E) none of these.
44. If $y = \sin(x^2 - 1)$ and $x = \sqrt{u^2 + 1}$, then $\frac{dy}{du}$ equals
- (A) $\frac{u \cos u^2}{\sqrt{1 + u^2}}$ (B) $\cos u^2$ (C) $\frac{\cos u^2}{2\sqrt{1 + u^2}}$
- (D) $2u \cos u^2$ (E) $\frac{u \cos(x^2 - 1)}{\sqrt{1 + u^2}}$.
45. If $f(x) = x^n$, n a positive integer, the first derivative of $f(x)$ which is identically zero is
- (A) the n th (B) the $(n - 1)$ st (C) the $(n + 2)$ nd
- (D) the first (E) the $(n + 1)$ st.

Practice Examination: 2

1. $\lim_{x \rightarrow \infty} \frac{20x^2 - 13x + 5}{5 - 4x^3}$ is
- (A) -5 (B) ∞ (C) 0 (D) 5 (E) 1.
2. $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos x}{x - \frac{\pi}{2}}$ is
- (A) -1 (B) 1 (C) 0 (D) ∞ (E) none of these.
3. $\lim_{x \rightarrow 0} x \sin \frac{1}{x}$ is
- (A) 1 (B) 0 (C) ∞ (D) -1 (E) none of these.
4. $\lim_{h \rightarrow 0} \frac{\ln(2 + h) - \ln 2}{h}$ is
- (A) 0 (B) $\ln 2$ (C) $\frac{1}{2}$ (D) $\frac{1}{\ln 2}$ (E) ∞ .

11. The equation of the tangent to the curve $y = e^x \ln x$ where $x = 1$ is

(A) $y = ex$ (B) $y = e^x + 1$ (C) $y = e(x - 1)$
 (D) $y = ex + 1$ (E) $y = x - 1$

- *12. If differentials are used for computation, then $\sqrt[3]{63}$ is approximately equal, to the nearest hundredth, to

(A) 4.00 (B) 3.98 (C) 3.93 (D) 3.80 (E) 3.88

13. The hypotenuse AB of a right triangle ABC is 5 ft and one leg AC is decreasing at the rate of 2 ft/sec. The rate in ft²/sec at which the area is changing when $AC = 3$ is

(A) $\frac{25}{4}$ (B) $\frac{7}{4}$ (C) $-\frac{3}{2}$ (D) $-\frac{7}{4}$ (E) $-\frac{7}{2}$

14. The derivative of a function f is given for all x by $f'(x) = x^2(x+1)^3(x-4)^2$. The set of x for which f is a relative maximum is

(A) $\{0, -1, 4\}$ (B) $\{-1\}$ (C) $\{0, 4\}$
 (D) $\{1\}$ (E) none of these.

15. The position of a point P on a line at time t is given by $s = t^3 + t^2 - t - 3$. P is moving to the right for

(A) $t > -1$ (B) $t < -\frac{1}{3}$ or $t > 1$
 (C) $t < -1$ or $t > \frac{1}{3}$ (D) $-1 < t < \frac{1}{3}$
 (E) $t < \frac{1}{3}$

16. If the displacement from the origin of a particle on a line is given by $s = 3 + (t - 2)^4$, then the number of times the particle reverses direction is

(A) 0 (B) 1 (C) 2 (D) 3 (E) none of these.

- *17. If a particle moves in a plane so that $x = 2 \cos 3t$ and $y = \sin 3t$, then its speed on the interval $0 \leq t < \frac{\pi}{2}$ is a maximum when t equals

(A) 0 (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{3}$ (E) $\frac{\pi}{2}$

18. The maximum value of the function $f(x) = xe^{-x}$ is

(A) $\frac{1}{e}$ (B) e (C) 1 (D) -1 (E) none of these.

19. A rectangle of perimeter 18 inches is rotated about one of its sides to generate a right circular cylinder. The rectangle which generates the cylinder of largest volume has area (in in.²) of

(A) 14 (B) 20 (C) $\frac{81}{4}$ (D) 18 (E) $\frac{77}{4}$

20. If $f'(x)$ exists on the closed interval $[a, b]$, then it follows that

(A) $f(x)$ is constant on $[a, b]$ NO
 (B) there exists a number c , $a < c < b$, such that $f'(c) = 0$ NO $f(a) \neq f(b)$
 (C) the function has a maximum value on the open interval (a, b)
 (D) the function has a minimum value on the open interval (a, b)
 (E) the mean value theorem applies. pg 176

21. $\int_1^2 (3x - 2)^3 dx$ is equal to

(A) $\frac{16}{3}$ (B) $\frac{63}{4}$ (C) $\frac{13}{3}$ (D) $\frac{85}{4}$ (E) none of these.

22. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^3 \alpha \cos \alpha d\alpha$ is equal to

(A) $\frac{3}{16}$ (B) $\frac{1}{8}$ (C) $-\frac{1}{8}$ (D) $-\frac{3}{16}$ (E) $\frac{3}{4}$

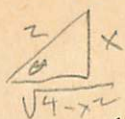
23. $\int x \cos x^2 dx$ equals

- (A) $\sin x^2 + C$ (B) $2 \sin x^2 + C$
 (C) $-\frac{1}{2} \sin x^2 + C$ (D) $\frac{1}{4} \cos^2 x^2 + C$
 (E) $\frac{1}{2} \sin x^2 + C$.

24. $\int_0^1 \frac{e^x}{(2 - e^x)^2} dx$ equals

- (A) $2 \ln(e - 2)$ (B) 1 (C) $\frac{1}{2 - e}$
 (D) $\frac{e - 1}{2 - e}$ (E) none of these.

*25. If we let $\frac{x}{2} = \sin \theta$, then $\int_0^2 \frac{x^2 dx}{\sqrt{4 - x^2}}$ is equivalent to



(A) $4 \int_0^1 \sin^2 \theta d\theta$

(B) $\int_0^{\frac{\pi}{2}} 4 \sin^2 \theta d\theta$

(C) $\int_0^{\frac{\pi}{2}} 2 \sin \theta \tan \theta d\theta$

(D) $\int_0^2 \frac{2 \sin^2 \theta}{\cos^2 \theta} d\theta$

(E) $4 \int_{\frac{\pi}{2}}^0 \sin^2 \theta d\theta$.

26. $\int \frac{x^2 + x - 1}{x^2 - x} dx$ equals

- (A) $x + 2 \ln |x^2 - x| + C$ (B) $\ln |x| + \ln |x - 1| + C$
 (C) $1 + \ln |x^2 - x| + C$ (D) $x + \ln |x^2 - x| + C$
 (E) $x - \ln |x| - \ln |x - 1| + C$.

27. $\int_{-1}^1 (1 - |x|) dx$ equals

- (A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) none of these.

$\frac{1}{2} \int \cos u du =$

$\frac{1}{2} \sin u \Rightarrow \frac{1}{2} \sin x^2$

Let $u = 2 - e^x$
 $du = -e^x dx$
 $x=0 \Rightarrow u=1$
 $x=1 \Rightarrow 2-e=u$

$-\int_1^{2-e} \frac{du}{u^2} = \int_{2-e}^1 \frac{du}{u^2}$

across 0! undefined

as $x \rightarrow 1$, $e^x \rightarrow e$
 on its way $2 - e^x$ will be zero.

$\int \frac{4 \sin^2 \theta}{2 \cos \theta} d\theta$

$x=0 \Rightarrow \theta=0$
 $x=2 \Rightarrow \theta=\frac{\pi}{2}$

$\int dx + \int \frac{du}{u}$
 $x + \ln |x^2 - x| + C$

$\frac{x^2 - x}{x^2 - x - 1}$
 $\frac{x^2 - x - 1}{2x - 1}$

$\int_{-1}^0 1 + x dx + \int_0^1 1 - x dx = x + \frac{x^2}{2} \Big|_{-1}^0 + x - \frac{x^2}{2} \Big|_0^1$

$1 - \frac{1}{2} + 1 - \frac{1}{2} = 1$

28. $\int_{-1}^0 e^{-x} dx$ equals $-e^{-x} \Big|_{-1}^0 = -1 + e$

(A) $1 - e$ (B) $\frac{1 - e}{e}$ (C) $e - 1$

(D) $1 - \frac{1}{e}$ (E) $e + 1$.

29. The general solution of the differential equation $\frac{dy}{dx} = \frac{1 - 2x}{y}$

is a family of

(A) straight lines (B) circles (C) hyperbolas

(D) parabolas (E) ellipses.

30. If $F'(x) = G'(x)$ for all x and k is a constant, then it is necessary that

(A) $F(x) = G(x) + k$

(B) $F(x) = G(x)$ NO

(C) $F(k) = G(k)$ NO

(D) $F(x) = kG(x)$ possible NO

(E) $F(x) = G(x + k)$.

31. The area enclosed by the curve $y^2 = x^3$ and the line segment joining (1,1) and (4,-8) is given by

(A) $2 \int_0^1 x^{3/2} dx + \int_1^4 (-3x + 4 - x^{3/2}) dx$

(B) $2 \int_0^1 x^{3/2} dx + \int_1^4 (4 - 3x + x^{3/2}) dx$

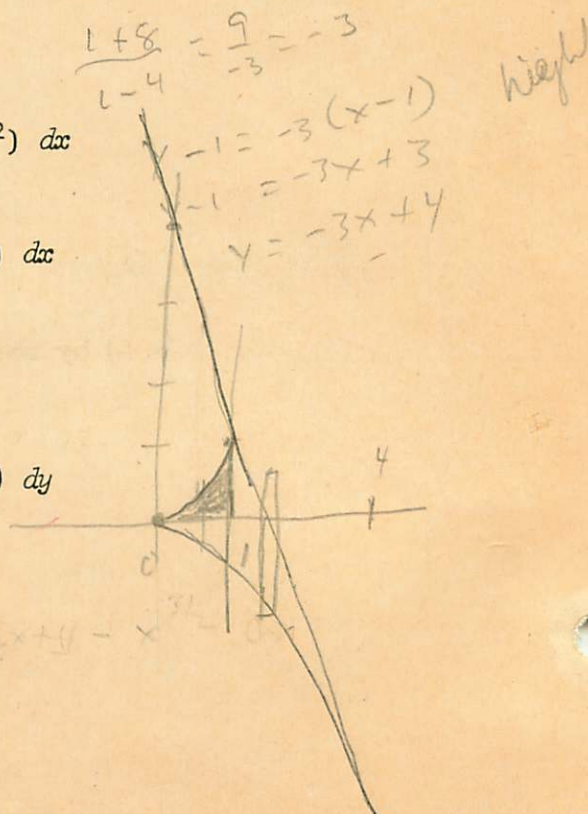
(C) $\int_0^4 (4 - 3x - x^{3/2}) dx$

(D) $2 \int_0^1 x^{3/2} dx + \int_{-8}^1 \left(\frac{4-y}{3} - y^{2/3} \right) dy$

(E) $\int_1^4 (x^{3/2} + 3x - 4) dx$.

Since $y^2 = x^3$ we know, symmetric w.r.t. x-axis.

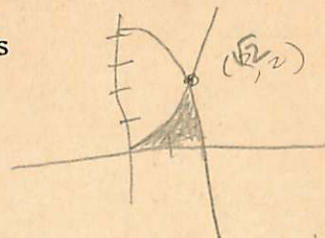
$2 \int_0^1 x^{3/2} dx + \int_1^4 (-3x + 4 - (-x^{3/2})) dx$
 ↑ above x-axis, Top Curve
 ↑ below x-axis, Bottom Curve



32. The first quadrant area bounded below by the x -axis and laterally

by the curves of $y = x^2$ and $y = 4 - x^2$ equals

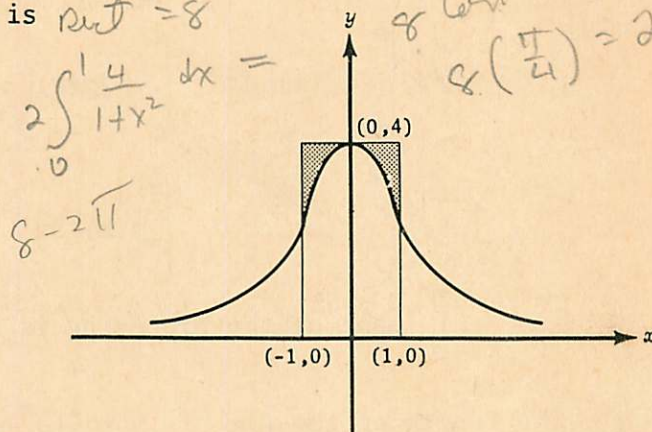
- (A) $6 - 2\sqrt{3}$ (B) $\frac{10}{3}$ (C) $\frac{8}{3}(2 - \sqrt{2})$
 (D) $\frac{4\sqrt{2}}{3}$ (E) none of these.



33. The equation of the curve shown is

$y = \frac{4}{1+x^2}$. The area of the shaded region of the rectangle equals

- (A) $4 - \frac{\pi}{4}$ (B) $8 - 2\pi$
 (C) $8 - \pi$ (D) $8 - \frac{\pi}{2}$
 (E) $2\pi - 4$.



- *34. A curve is given parametrically by the equations $x = t$, $y = 1 - \cos t$.

The area bounded by the curve and the x -axis on the interval $0 \leq t \leq 2\pi$ is equal to

- (A) $2(\pi + 1)$ (B) π (C) 4π (D) $\pi + 1$ (E) 2π .

- *35. The volume of an "inner tube" with inner diameter 4 ft and outer diameter 8 ft is, in ft^3 ,

- (A) $4\pi^2$ (B) $12\pi^2$ (C) $8\pi^2$ (D) $24\pi^2$ (E) $6\pi^2$.

- *36. The volume generated by rotating the region bounded by $y = \frac{1}{\sqrt{x}}$, $x = 1$, $x = 4$, and $y = 0$ about the y -axis is

- (A) $\frac{28\pi}{3}$ (B) $\pi \ln 4$ (C) $\frac{64\pi}{3}$ (D) $\frac{14\pi}{3}$ (E) $\frac{16\pi}{3}$

- *37. If $f(x)$ and $g(x)$ are both continuous functions on the closed interval $[a, b]$, if $f(a) = g(a)$ and $f(b) = g(b)$, and if,

further, $f(x) > g(x)$ for all x , $a < x < b$, then it follows that

(A) $\int_a^b f(x) dx \geq 0$

Possible. (B) $\int_a^b |g(x)| dx > \int_a^b |f(x)| dx$

Possible. (C) $\int_a^b |f(x)| dx > \int_a^b |g(x)| dx$

(D) $\int_a^b f(x) dx > \int_a^b g(x) dx$

(E) none of the preceding is necessary.

*38. The length of the arc of $y = \frac{1}{2}(e^x + e^{-x})$ from $x = 0$ to $x = 1$ equals

(A) $\frac{1}{2}(e + e^{-1})$

(B) $\frac{1}{2}(e - e^{-1})$

(C) $e - \frac{1}{e}$

(D) $e + \frac{1}{e} - 2$

(E) $\frac{1}{2}(e + \frac{1}{e}) - 1$

*39. The area of the surface of revolution generated if one arch of the cycloid $x = \theta - \sin \theta$, $y = 1 - \cos \theta$ is rotated about the x -axis is given by the integral

(A) $4\pi \int_0^{2\pi} \sin \frac{\theta}{2} d\theta$

(B) $2\sqrt{2}\pi \int_0^{\pi} (1 - \cos \theta)^{1/2} d\theta$

(C) $2\sqrt{2}\pi \int_0^{2\pi} (1 - \cos \theta)^{3/2} d\theta$

(D) $2\sqrt{2}\pi \int_0^{2\pi} (\theta - \sin \theta) \sqrt{1 - \cos \theta} d\theta$

(E) none of these.

Tricky 40. A particle moves along a line with velocity (in ft/sec) $v = t^2 - t$.

The total distance (in ft) travelled from $t = 0$ to $t = 2$ equals

(A) $\frac{1}{3}$

(B) $\frac{2}{3}$

(C) 2

(D) $\frac{1}{3}$

(E) $\frac{4}{3}$

$$\int_0^2 t^2 - t \, dt = \left[\frac{t^3}{3} - \frac{t^2}{2} \right]_0^2 = \frac{8}{3} - 2 = \frac{2}{3}$$

*41. $\lim_{n \rightarrow \infty} \frac{\sqrt{1} + \sqrt{2} + \dots + \sqrt{n}}{n^{3/2}}$ is equal to the definite integral

(A) $\int_0^1 \sqrt{x} \, dx$

(B) $\int_0^1 \frac{1}{\sqrt{x}} \, dx$

(C) $\int_1^2 \sqrt{x} \, dx$

(D) $\int_0^1 \frac{1}{x} \, dx$

(E) none of these.

*42. A force of 20 lb compresses a spring from its natural length of 30 inches to 26 inches. The work done (in inch-lb) in compressing it from 26 inches to 22 inches is equal to

- (A) 20 (B) 40 (C) 80 (D) 120 (E) 240 .

43. The lines $x - ky = 3$ and $kx + y = 4$ are perpendicular for

- (A) no real k (B) $k = 0$ only (C) $k = 1$ only

- (D) $k = -1$ only (E) all real k .

44. The cartesian equation of the polar curve $r^2 = 2\sin \theta + 2\cos \theta$ is

(A) $(x - 1)^2 + (y - 1)^2 = 2$

(B) $x^2 + y^2 = 2$

(C) $x + y = 2$

(D) $x^2 + y^2 = 4$

(E) $y^2 - x^2 = 4$.

45. If x and y are real numbers, the domain of the function

$f(x) = \frac{\sqrt{x^2 + 1}}{4 - x}$ is

- (A) all x except $x = 2$ or $x = -2$

- (B) all x except $x = 4$

- (C) $|x| \leq 1$

- (D) $x > 1$ or $x < -1$

- (E) all x .

1. $\lim_{x \rightarrow 2} \frac{x^2 - 2}{4 - x^2}$ is *Doesn't reduce out of the problem*
 (A) -2 (B) -1 (C) $-\frac{1}{2}$ (D) 0 (E) nonexistent.

2. $\lim_{x \rightarrow \infty} \frac{\sqrt{x} - 4}{4 - 3\sqrt{x}}$ is *$1 - \frac{4}{\sqrt{x}} = -\frac{1}{3}$*
 (A) $-\frac{1}{3}$ (B) $-\frac{1}{4}$ (C) ∞ (D) 0 (E) $\frac{1}{3}$.

3. $\lim_{x \rightarrow 0} \frac{\sin^2 \frac{x}{2}}{x^2}$ is *$\frac{\sin \frac{x}{2}}{2 \frac{x}{2}} \cdot \frac{\sin \frac{x}{2}}{2 \frac{x}{2}} = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$*
 (A) 4 (B) 0 (C) $\frac{1}{4}$ (D) 2 (E) nonexistent.

4. $\lim_{x \rightarrow 0} \frac{e^x - 1}{x}$ is *$\frac{e^x}{1} \Rightarrow 1$ l'Hopital's rule*
 (A) 1 (B) e (C) $\frac{1}{e}$ (D) 0 (E) nonexistent.

5. The slope of $y = |x|$ at the point where $x = \frac{1}{2}$ is *$y = x$ since we are interested where $x = \frac{1}{2}$ and $x > 0$*
 (A) -1 (B) 0 (C) $\frac{1}{2}$ (D) 1 (E) nonexistent.

6. If $y = \frac{e^{\ln u}}{u}$ then $\frac{dy}{du}$ equals *$y = \frac{u}{u} = 1$ $\frac{dy}{dx} = 0$*
 (A) $\frac{e^{\ln u}}{u^2}$ (B) $e^{\ln u}$ (C) $\frac{2e^{\ln u}}{u^2}$ (D) 1 (E) 0.

7. If $y = \sin^3(1 - 2x)$ then $\frac{dy}{dx}$ is *$(-2)3 \sin^2(1 - 2x) \cos(1 - 2x)$*
 (A) $3 \sin^2(1 - 2x)$ (B) $-2 \cos^3(1 - 2x)$
 (C) $-6 \sin^2(1 - 2x)$ (D) $-6 \sin^2(1 - 2x) \cos(1 - 2x)$
 (E) $-6 \cos^2(1 - 2x)$.

8. If $f(u) = \tan^{-1} u^2$ and $g(u) = e^u$ then the derivative of $f(g(u))$ is *$f(e^u) = \frac{\tan^{-1} e^{2u}}{1 + e^{4u}}$*
 (A) $\frac{2ue^u}{1 + u^4}$ (B) $\frac{2ue^{u^2}}{1 + u^4}$ (C) $\frac{2e^u}{1 + 4e^{2u}}$
 (D) $\frac{2e^{2u}}{1 + e^{4u}}$ (E) $\frac{2e^{2u}}{\sqrt{1 - e^{4u}}}$.

9. The equation of the tangent to the curve of $xy - x + y = 2$ at the point where $x = 0$ is *$y = 2$*
 (Choices on next Page)

(A) $y = -x$ (B) $y = \frac{1}{2}x + 2$ (C) $y = x + 2$

(D) $y = 2$ (E) $y = 2 - x$.

10. If $y = \frac{1}{x}$ then $y^{iv}(1)$ equals

(A) $-4!$ (B) $-3!$ (C) $4!$ (D) $5!$ (E) $3!$.

11. If $y = x^2 e^{1/x}$ ($x \neq 0$), then $\frac{dy}{dx}$ is

(A) $x e^{1/x}(x+2)$ (B) $e^{1/x}(2x-1)$ (C) $\frac{-2e^{1/x}}{x}$

(D) $e^{-x}(2x-x^2)$ (E) none of these.

*12. If differentials are used for computation, then $\sqrt[3]{127}$ is approximately equal to

(A) 5.27 (B) 5.01 (C) 5.03 (D) 5.10 (E) 5.07.

13. A point moves along the curve $y = x^2 + 1$ so that the x -coordinate

is increasing at the constant rate of $\frac{3}{2}$ units per second. The rate

in units per second at which the distance from the origin is

changing when the point has coordinates (1,2) is equal to

(A) $\frac{7\sqrt{5}}{10}$ (B) $\frac{3\sqrt{5}}{2}$ (C) $3\sqrt{5}$ (D) $\frac{15}{2}$ (E) $\sqrt{5}$.

14. The set of x for which the curve of $y = 1 - 6x^2 - x^4$ has

inflection points is

(A) $\{0\}$ (B) $\{\pm\sqrt{3}\}$ (C) $\{1\}$ (D) $\{\pm 1\}$

(E) none of these.

15. If the position of a particle on a line at time t is given by

$s = t^3 + 3t$, then the speed of the particle is decreasing when

(A) $-1 < t < 1$ (B) $-1 < t < 0$ (C) $t < 0$

(D) $t > 0$ (E) $|t| > 1$.

- *16. A particle moves along the parabola $x = 3y - y^2$ so that $\frac{dy}{dt} = 3$

at all time t . The speed of the particle when it is at position

(2,1) is equal to

- (A) 0 (B) 3 (C) $\sqrt{13}$ (D) $3\sqrt{2}$ (E) none of these.

$\frac{d^2y}{dx^2} = \frac{x''y' - y''x'}{(x')^3}$ really is application of quotient rule of $\frac{d(\frac{y}{x})}{dx}$

- *17. The motion of a particle in a plane is given by the pair of

equations $x = e^t \cos t$, $y = e^t \sin t$. The magnitude of

its acceleration at any time t equals

- (A) $\sqrt{x^2 + y^2}$ (B) $2e^t \sqrt{\cos 2t}$ (C) $2e^t$

- (D) e^t (E) $2e^{2t}$

18. A rectangle with one side on the x -axis is inscribed in the triangle

formed by the lines $y = x$, $y = 0$, and $2x + y = 12$. The area of the largest such rectangle is

- (A) 6 (B) 3 (C) $\frac{5}{2}$ (D) 5 (E) 7

19. The abscissa of the first-quadrant point which is on the curve of $x^2 - y^2 = 1$ and closest to the point (3,0) is

- (A) 1 (B) $\frac{3}{2}$ (C) 2 (D) 3 (E) none of these.

20. If c represents the number defined by Rolle's Theorem, then for

the function $f(x) = x^3 - 3x^2$ on the interval $0 \leq x \leq 3$,

c is equal to

- (A) 2 (B) 1 (C) 0 (D) $\sqrt{2}$ (E) none of these.

21. $\int \frac{x dx}{\sqrt{9-x^2}}$ equals

- (A) $-\frac{1}{2} \ln \sqrt{9-x^2} + C$ (B) $\sin^{-1} \frac{x}{3} + C$

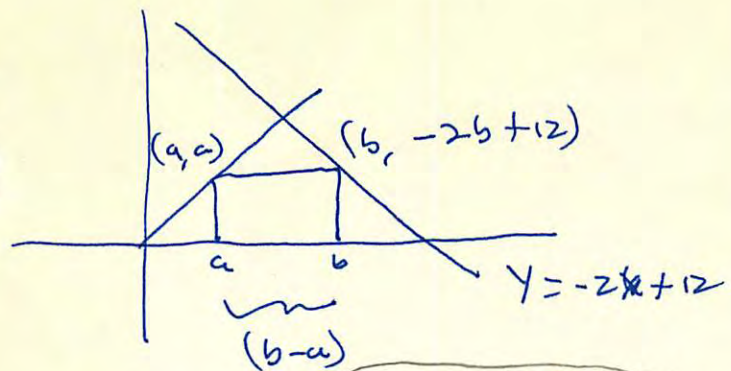
- (C) $-\sqrt{9-x^2} + C$

- (D) $-\frac{1}{4}\sqrt{9-x^2} + C$

- (E) $2\sqrt{9-x^2} + C$

#3

(18)



$$A = (b-a)a$$

Set $a = -2b + 12$

$$b-a \Rightarrow b - (-2b+12)$$

$$(b-a) = 3b-12 = 3(b-4)$$

$$a = -2b + 12$$

$$2b = 12 - a$$

$$b = 6 - \frac{1}{2}a$$

$$b-a = 6 - \frac{1}{2}a - a$$

$$= 6 - \frac{3}{2}a$$

$$A =$$

$$A = (6 - \frac{3}{2}a)a$$

$$6a - \frac{3}{2}a^2$$

$$A' = 6 - 3a \stackrel{\text{Set}}{=} 0$$

$$a = 2$$

$$A = 3(b-4)(12-2b)$$

$$A' = -6(b-4)(2b-12)$$

$$-6(b-4)(b-6)$$

$$= -6(b^2 - 10b + 24)$$

$$A' = -6(2b-10) \stackrel{\text{Set}}{=} 0$$

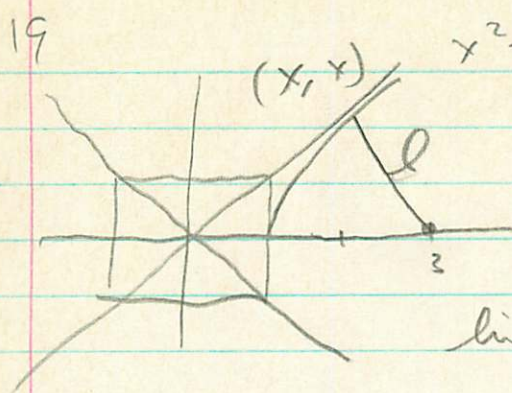
$$b = 5.$$

$$A'' = -12 < 0 \therefore \text{max at } b=5.$$

$$b=5 \Rightarrow a=2$$

$$A = 2(5-2) = 6.$$

19



$$x^2 - y^2 = 1$$

$$2x - 2y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{x}{y} = m \text{ we want } -\frac{y}{x}$$

line l has equation

$$y = -\frac{y}{x}(x-3)$$

$$y = -y + \frac{3y}{x}$$

$$2 = \frac{3}{x}$$

$$x = \frac{3}{2}$$

$$\text{Let } u = \tan x \\ du = \sec^2 x \, dx$$

$$\int u^2 du = \frac{1}{3} u^3 \Rightarrow \frac{1}{3} \tan^3 x \Big|_{\frac{\pi}{4}}^{\frac{\pi}{3}} =$$

$$\frac{1}{3} [3\sqrt{3} - 1] = \sqrt{3} - \frac{1}{3}$$

*22. $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \sec^2 x \tan^2 x \, dx$ equals

- (A) 5 (B) $\sqrt{3} - 1$ (C) $\frac{8}{3} - \frac{2\sqrt{2}}{3}$ (D) $\sqrt{3}$ (E) $\sqrt{3} - \frac{1}{3}$.

23. $\int \frac{(y-1)^2}{2y} dy$ equals

- (A) $\frac{y^2}{4} - y + \frac{1}{2} \ln |y| + C$ (B) $y^2 - y + \ln |2y| + C$

- (C) $y^2 - 4y + \frac{1}{2} \ln |2y| + C$ (D) $\frac{(y-1)^3}{3y^2} + C$

- (E) $\frac{1}{2} - \frac{1}{2y^2} + C$.

*24. $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cot x \, dx$ equals

- (A) $\ln \frac{1}{2} \Rightarrow$ (B) $\ln 2$ (C) $-\ln(2 - \sqrt{3})$

- (D) $\ln(\sqrt{3} - 1)$ (E) none of these.

*25. $\int_1^e \ln x \, dx$ equals

- (A) $\frac{1}{2}$ (B) $e - 1$ (C) $e + 1$ (D) 1 (E) -1 .

26. If $F(x) = \int_1^x \sqrt{t^2 + 3t} \, dt$ then $F'(x)$ equals

- (A) $\frac{2}{3}[(x^2 + 3x)^{3/2} - 8]$ (B) $\sqrt{x^2 + 3x}$

- (C) $\sqrt{x^2 + 3x} - 2$ (D) $\frac{1}{2} \frac{(2x + 3)}{\sqrt{x^2 + 3x}}$ (E) none of these.

*27. Which one of the following improper integrals converges?

(A) $\int_{-1}^1 \frac{dx}{(x+1)^2}$

(B) $\int_1^{\infty} \frac{dx}{\sqrt{x}}$

(C) $\int_0^{\infty} \frac{dx}{(x^2 + 1)}$

(D) $\int_1^3 \frac{dx}{(2-x)^3}$

- (E) none of these.

28. If $f(x)$ is continuous on the interval $-a \leq x \leq a$ then it

follows that $\int_{-a}^a f(x) dx$

(A) equals $2 \int_0^a f(x) dx$

(B) equals 0

(C) equals $f(a) - f(-a)$

(D) represents the total

area bounded by the curve $y = f(x)$, the x -axis, and

the vertical lines $x = -a$ and $x = a$

→ (E) is not necessarily any of these.

29. If $\frac{dy}{dx} = \frac{y}{x}$ ($x > 0, y > 0$) and $y = 3$ when $x = 1$, then

(A) $x^2 + y^2 = 10$

(B) $y = x + \ln 3$

(C) $y^2 - x^2 = 8$

(D) $y = 3x$

(E) $y^2 - 3x^2 = 6$

30. The area bounded by the cubic $y = x^3 - 3x^2$ and the line $y = -4$ is given by the integral

(A) $\int_{-1}^2 (x^3 - 3x^2 + 4) dx$

(B) $\int_{-1}^2 (x^3 - 3x^2 - 4) dx$

(C) $2 \int_0^2 (x^3 - 3x^2 - 4) dx$

(D) $\int_{-1}^2 (4 - x^3 + 3x^2) dx$

(E) $\int_0^3 (3x^2 + x^3) dx$

31. The area bounded by the curve $y = \frac{1}{x+1}$, the axes, and the line $x = e - 1$ equals

(A) $1 - \frac{1}{e^2}$

(B) $\ln(e - 1)$

(C) 1

(D) 2

(E) $2\sqrt{e+1} - 2$

32. The area bounded by the curve $x = 3y - y^2$ and the line $x = -y$ is represented by

(3)

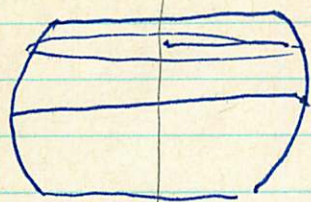
#34

Circles

radius

$y=1$

$y=0$



$$2\pi \int_0^1 (\sqrt{4-y^2})^2 dy$$

$$x^2 + y^2 = 4$$

$$y = \sqrt{4-x^2}$$

$$x = \sqrt{4-y^2}$$

$$2\pi \int_0^1 4-y^2 dy$$

$$2\pi \left(4y - \frac{y^3}{3} \right) \Big|_0^1$$

$$2\pi \left[8 - \frac{8}{3} \right]$$

$$2\pi \left(\frac{24-8}{3} \right)$$

$$\frac{32\pi}{3}$$

$$2\pi \left(4 - \frac{1}{3} \right)$$

$$2\pi \left(\frac{12-1}{3} \right)$$

$$\frac{22\pi}{3}$$

$(2)(\pi) \int_0^2 (\sqrt{4-y^2})^2 dy = \text{Top \& Bottom parts removed, full sphere } \frac{4}{3}\pi r^3 = \frac{32\pi}{3}$
 $\frac{32\pi}{3} - \frac{10\pi}{3} = \frac{22\pi}{3}$

- (A) $\int_0^4 (2y - y^2) dy$ (B) $\int_0^4 (4y - y^2) dy$
 (C) $\int_0^3 (3y - y^2) dy + \int_0^4 y dy$ (D) $\int_0^3 (y^2 - 4y) dy$
 (E) $\int_0^3 (2y - y^2) dy$

$x = 2a \tan \theta$
 $0 \leq \theta \leq \frac{\pi}{4}$

$\theta = \frac{\pi}{4}$
 $2a \cos^2 \theta \cdot 2a \sin^2 \theta d\theta$
 $\int_0^{\frac{\pi}{4}} 4a^2 \sin^2 \theta \cos^2 \theta d\theta$
 $4a^2 \int_0^{\frac{\pi}{4}} \frac{1 - \cos 4\theta}{8} d\theta$
 $\frac{4a^2}{8} \left[\theta - \frac{\sin 4\theta}{4} \right]_0^{\frac{\pi}{4}} = \frac{\pi a^2}{4}$

*33. The area bounded by the curve with parametric equations $x = 2a \tan \theta$, $y = 2a \cos^2 \theta$, and the lines $x = 0$ and $x = 2a$ is equal to

- (A) πa^2 (B) $2\pi a^2$ (C) $\frac{\pi a}{4}$ (D) $\frac{\pi a}{2}$ (E) none of these.

See top of page 34.

34. A solid is cut out of a sphere of radius 2 by two parallel planes each 1 unit from the center. The volume of this solid is

- (A) 8π (B) $\frac{32\pi}{3}$ (C) $\frac{25\pi}{3}$ (D) $\frac{22\pi}{3}$ (E) $\frac{20\pi}{3}$

35. The volume generated by rotating the region bounded by

$y = \frac{1}{\sqrt{1+x^2}}$, $y = 0$, $x = 0$, and $x = 1$ about the y -axis is

- (A) $\frac{\pi^2}{4}$ (B) $\pi\sqrt{2}$ (C) $2\pi(\sqrt{2} - 1)$ (D) $\frac{2\pi}{3}(2\sqrt{2} - 1)$
 (E) $1 + \frac{\pi^2}{4}$

shell
 $\int_0^1 2\pi x \cdot \frac{1}{\sqrt{1+x^2}} dx$

36. The region bounded by $y = e^x$, $y = 1$, and $x = 2$ is rotated about

the x -axis. The volume of the solid generated is given by the integral:

- (A) $\pi \int_0^2 e^{2x} dx$ (B) $2\pi \int_1^{e^2} (2 - \ln y)(y - 1) dy$
 (C) $\pi \int_0^2 (e^{2x} - 1) dx$ (D) $2\pi \int_0^2 y(2 - \ln y) dy$
 (E) $\pi \int_0^2 (e^x - 1)^2 dx$

washer



cross-section πe^{2x}
 minus the cylinder $\pi 1^2$ so
 $\pi \int_0^2 (e^{2x} - 1) dx$

37. A particle moves on a straight line so that its velocity at time t is given by $v = 4s$, where s is its distance from the origin.

If $s = 3$ when $t = 0$, then when $t = \frac{1}{2}$, s equals

- (A) $1 + e^2$ (B) $2e^3$ (C) e^2 (D) $2 + e^2$ (E) $3e^2$.

- *38. The length of the arc of $y = \frac{1}{2}x^2 - \frac{1}{4}\ln x$ from $x = 1$ to $x = 4$ is

- (A) $\frac{15}{2} + \ln 2$ (B) $\frac{1}{2}(15 + \ln 2)$ (C) $\frac{1}{2}(17 + \ln 2)$
(D) $3\frac{15}{64}$ (E) $\frac{45}{16}$.

39. The average (mean) value of $y = (x - 3)^2$ over the interval from $x = 1$ to $x = 3$ equals

- (A) 2 (B) $\frac{2}{3}$ (C) $\frac{4}{3}$ (D) $\frac{8}{3}$ (E) none of these.

- *40. $\lim_{n \rightarrow \infty} [\ln(1 + \frac{1}{n}) + \ln(1 + \frac{2}{n}) + \dots + \ln(1 + \frac{n}{n})] \cdot \frac{1}{n}$ is equal to the definite integral

- (A) $\int_0^1 \ln x \, dx$ (B) $\int_1^2 \ln \frac{1}{x} \, dx$ (C) $\int_1^2 \ln(1 + x) \, dx$
(D) $\int_1^2 \ln x \, dx$ (E) none of these.

41. A particle moves along a line so that its acceleration, a , at time t is $a = -t^2$. If the particle is at the origin when $t = 0$ and 3 units to the right of the origin when $t = 1$, then its velocity at $t = 0$ is

- (A) 0 (B) $\frac{1}{12}$ (C) $2\frac{11}{12}$ (D) $\frac{37}{12}$ (E) none of these.

42. The curve with parametric equations $x = \sqrt{t - 2}$ and $y = \sqrt{6 - t}$ is

- (A) part of a circle (B) a parabola (C) a straight line
(D) part of a hyperbola (E) none of these.

$$\frac{3.5 + 2(-4) - 7}{\sqrt{9+4}} = \frac{15-15}{\sqrt{13}}$$

43. The distance from $(-4, 5)$ to the line $3y + 2x - 7 = 0$ is

- (A) $\frac{9}{5}$ (B) $\frac{9}{\sqrt{13}}$ (C) 1 (D) $\sqrt{13}$ (E) none of these.

goes through this pt.

dist is zero

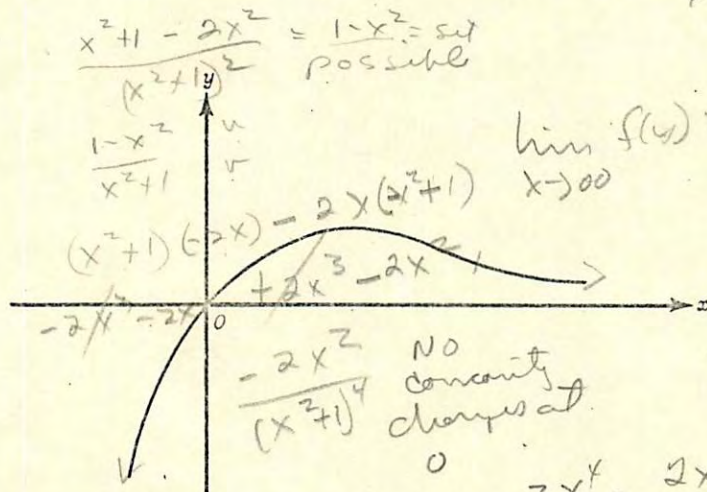
44. The set of x for which $|x + 1| > 2$ and $|x - 1| \leq 2$ is

- (A) the null set (B) $x < -3$ or $x > -1$
(D) $x < -3$ or $x \geq -1$ (E) $1 < x \leq 3$

$$1 < x \leq 3$$

45. Which of the following functions could have the graph sketched below?

- (A) $f(x) = xe^x$ (B) $f(x) = xe^{-x}$ (C) $f(x) = \frac{e^x}{x}$ No $x \neq 0$
(D) $f(x) = \frac{x}{x^2 + 1}$ (E) $f(x) = \frac{x^2}{x^3 + 1}$



$$f' = xe^x + e^x$$

critical $x = -1$
no max is > 0
in graph

$$f' = -xe^x + e^{-x}$$

$$f'' = xe^{-x} - e^{-x} - e^{-x}$$

change of concavity at $x = 2$

$$2x^4 + 2x - 3x^2 = 2x - x^4 \quad x^3(2 - x^3)$$

Practice Examination: 4

1. $\lim_{x \rightarrow 0} \frac{x^3 - 3x^2}{x}$ is $\lim_{x \rightarrow 0} \frac{x^2 - 3x}{1} = 0$ (Correct) $\downarrow$
- (A) 0 (B) ∞ (C) -3 (D) 1 (E) 3
2. $\lim_{h \rightarrow 0} \frac{\sin(\frac{\pi}{2} + h) - 1}{h}$ is $\text{let } y = \sin x \quad y' = \cos x \quad \cos x|_{\frac{\pi}{2}} = 0$
 $x = \frac{\pi}{2}$
- (A) 1 (B) -1 (C) 0 (D) ∞ (E) none of these.

#6

$$\frac{dy}{dx} = \frac{x}{\sqrt{x^2+16}}$$

$$\frac{\sqrt{x^2+16} - \frac{x^2}{\sqrt{x^2+16}}}{x^2+16}$$

$$\frac{x^2+16-x^2}{(x^2+16)\sqrt{x^2+16}}$$

$$\frac{16}{(x^2+16)^{3/2}}$$

3. $\lim_{x \rightarrow 2} [x]$ (where $[x]$ is the greatest integer in x) is

(A) 1 (B) 2 (C) 3 (D) ∞ (E) nonexistent.

4. $\lim_{x \rightarrow \infty} x \tan \frac{\pi}{x}$ is $\lim_{x \rightarrow \infty} \pi \frac{\sin \frac{\pi}{x}}{\frac{\pi}{x}} \cdot \frac{1}{\cos \frac{\pi}{x}} =$

(A) 0 (B) 1 (C) $\frac{1}{\pi}$ (D) π (E) ∞ .

5. If $y = \ln \frac{x}{\sqrt{x^2 + 1}}$ then $\frac{dy}{dx}$ is

(A) $\frac{1}{x^2 + 1}$ (B) $\frac{1}{x(x^2 + 1)}$ (C) $\frac{2x^2 + 1}{x(x^2 + 1)}$

(D) $\frac{1}{x\sqrt{x^2 + 1}}$ (E) $\frac{1 - x^2}{x(x^2 + 1)}$.

6. If $y = \sqrt{x^2 + 16}$, then $\frac{d^2y}{dx^2}$ is

(A) $-\frac{1}{4(x^2 + 16)^{3/2}}$ (B) $4(3x^2 + 16)$

(C) $\frac{16}{\sqrt{x^2 + 16}}$ (D) $\frac{2x^2 + 16}{(x^2 + 16)^{3/2}}$ (E) $\frac{16}{(x^2 + 16)^{3/2}}$.

7. If $x = a \cot \theta$ and $y = a \sin^2 \theta$, then $\frac{dy}{dx}$, when $\theta = \frac{\pi}{4}$, is

equal to $\frac{dy}{dx} = -a \csc^2 \theta \cdot \frac{dy}{d\theta} = -a \csc^2 \theta \cdot 2a \sin \theta \cos \theta = -2 \sin \theta \cos \theta$

(A) $\frac{1}{2}$ (B) -1 (C) 2 (D) $-\frac{1}{2}$ (E) $-\frac{1}{4}$.

8. The equation of the tangent to the curve $2x^2 - y^4 = 1$ at the point $(-1, 1)$ is

(A) $y = -x$ (B) $y = 2 - x$ (C) $4y + 5x + 1 = 0$

(D) $x - 2y + 3 = 0$ (E) $x - 4y + 5 = 0$.

9. If $f(x) = x \ln x$ then $f'''(e)$ equals

(A) $\frac{1}{e}$ (B) 0 (C) $-\frac{1}{e^2}$ (D) $\frac{1}{e^2}$ (E) $\frac{2}{e^3}$.

$$15 - 8 - 7 = 0$$

43. The distance from $(-4, 5)$ to the line $3y + 2x - 7 = 0$ is

goes through the pt

- (A) $\frac{9}{5}$ (B) $\frac{9}{\sqrt{13}}$ (C) 1 (D) $\sqrt{13}$ (E) none of these.

44. The set of x for which $|x + 1| > 2$ and $|x - 1| \leq 2$ is

- (A) the null set (B) $x < -3$ or $x > -1$
(D) $x < -3$ or $x \geq -1$ (E) none of these.
(C) $1 < x \leq 3$

45. Which of the following functions could have the graph sketched below?

- (A) $f(x) = xe^x$ (B) $f(x) = xe^{-x}$ (C) $f(x) = \frac{e^x}{x}$
(D) $f(x) = \frac{x}{x^2 + 1}$ (E) $f(x) = \frac{x^2}{x^3 + 1}$

(12)

Should be!

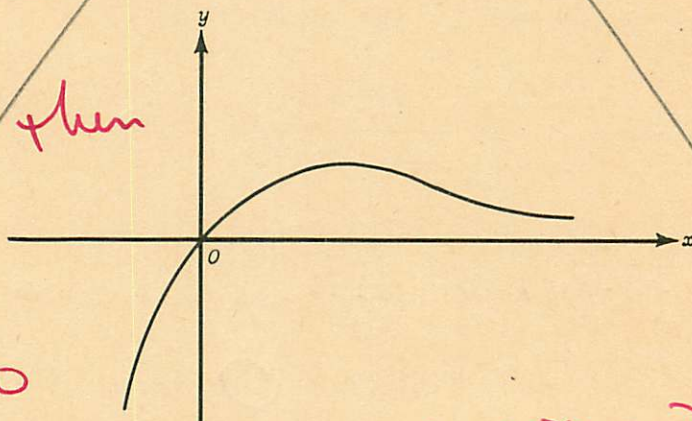
if $f(x) = \frac{e^{2x}}{x}$ then

$f'(0) = ?$

Since $f(x) = 1$

$f'(x) = 0$

$f'(0)$ DNE because $x=0$ is NOT $\in D_f$.



Practice Examination: 4

1. $\lim_{x \rightarrow 0} \frac{x^3 - 3x^2}{x}$ is

- (A) 0 (B) ∞ (C) -3 (D) 1 (E) 3.

2. $\lim_{h \rightarrow 0} \frac{\sin(\frac{\pi}{2} + h) - 1}{h}$ is

- (A) 1 (B) -1 (C) 0 (D) ∞ (E) none of these.

10. If $y = \sqrt{\ln(x^2 + 1)}$ ($x > 0$), then the derivative of y^2 with respect to $\ln x$ is equal to

(A) $\frac{2x}{x^2 + 1}$ (B) $\frac{2}{x^2 + 1}$ (C) $\frac{2x}{\ln x(x^2 + 1)}$
 (D) $\frac{2x^2}{x^2 + 1}$ (E) none of these.

11. If $f(t) = \frac{1}{t^2} - 4$ and $g(t) = \cos t$ then the derivative of

$f(g(t))$ is $f(\cos t) = \frac{1}{\cos^2 t} - 4$

(A) $2 \sec^2 t \tan t$ (B) $\tan t$ (C) $2 \sec t \tan t$
 (D) $\frac{2}{t^3 \sin t}$ (E) $-\frac{2}{\cos^3 t}$

12. If $f(x) = \frac{e^{\ln x}}{x}$ then $f(1)$ is

(A) 0 (B) 1 (C) e (D) $\frac{e}{2}$ (E) not defined.

13. A particle moves on a line according to the law $s = f(t)$ so that its velocity $v = ks$, where k is a nonzero constant.

Its acceleration is

(A) $k^2 v$ (B) $k^2 s$ (C) k (D) 0 (E) none of these.

- *14. The radius of a sphere is 10 in.; if the radius is increased by 0.01 in., then the approximate change, in in.³, in the volume is

(A) $.4\pi$ (B) $\frac{4}{9}\pi$ (C) 4π (D) 8π (E) 40π .

15. A relative maximum value of the function $y = \frac{\ln x}{x}$ is

(A) 1 (B) e (C) $\frac{2}{e}$ (D) $\frac{1}{e}$ (E) none of these.

16. If a particle moves on a line according to the law $s = t^5 + 2t^3$,

then the number of times it reverses direction is

(A) 4 (B) 3 (C) 2 (D) 1 (E) 0.

No. 17 *17. A particle moves counterclockwise on the circle $x^2 + y^2 = 25$ with constant speed of 2 ft/sec. Its velocity vector, $\mathbf{v}$, when the particle is at $(3, 4)$, equals

- (A) $-\frac{1}{5}(8\mathbf{i} - 6\mathbf{j})$ (B) $\frac{1}{5}(8\mathbf{i} - 6\mathbf{j})$ (C) $-2\sqrt{3}\mathbf{i} + 2\mathbf{j}$
 (D) $2\mathbf{i} - 2\sqrt{3}\mathbf{j}$ (E) $-2\sqrt{2}(\mathbf{i} - \mathbf{j})$.

18. A rectangular pig pen is to be built against a wall so that only three sides will require fencing. If p feet of fencing are to be used, the area of the largest possible pen is

- (A) $\frac{p^2}{2}$ (B) $\frac{p^2}{4}$ (C) $\frac{p^2}{8}$ (D) $\frac{p^2}{9}$ (E) $\frac{p^2}{16}$.

*19. Let $\mathbf{R} = a \cos kti + a \sin ktj$ be the (position) vector $x\mathbf{i} + y\mathbf{j}$ from the origin to a moving point $P(x, y)$ at time t , where a and k are positive constants. The acceleration vector, $\mathbf{a}$, equals

- (A) $-k^2\mathbf{R}$ (B) $a^2k^2\mathbf{R}$ (C) $-a\mathbf{R}$
 (D) $-ak^2(\cos t\mathbf{i} + \sin t\mathbf{j})$ (E) $-\mathbf{R}$.

20. The region bounded by the parabolas $y = x^2$ and $y = 6x - x^2$ is rotated about the x -axis so that a vertical line segment cut off

by the curves generates a ring. The value of x for which the ring of largest area is obtained is

- (A) 4 (B) 3 (C) $\frac{5}{2}$ (D) 2 (E) $\frac{3}{2}$.

21. $\int \frac{\cos x}{4 + 2 \sin x} dx$ equals

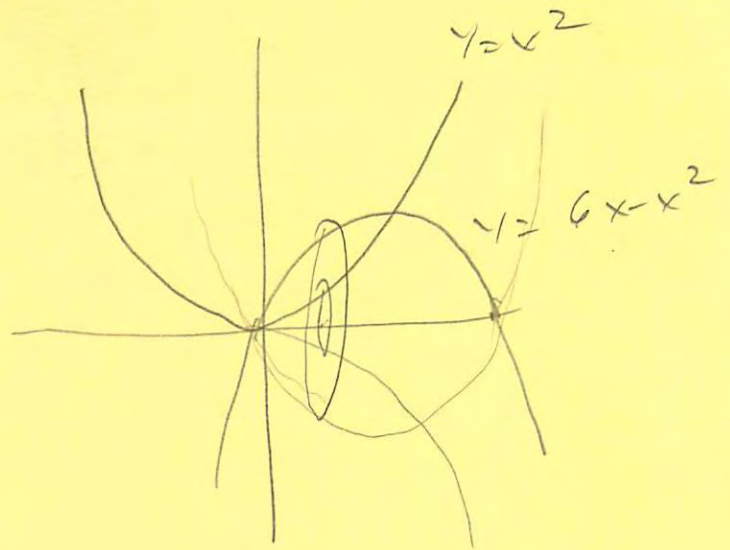
- (A) $\sqrt{4 + 2 \sin x} + C$ (B) $-\frac{1}{2(4 + \sin x)} + C$
 (C) $\ln \sqrt{4 + 2 \sin x} + C$ (D) $2 \ln |4 + 2 \sin x| + C$

- (E) $\frac{1}{4} \sin x - \frac{1}{2} \csc^2 x + C$.

MG 4 #20

$$y = x^2$$

$$y = 6x - x^2$$



$$A(x) = \pi [(6x - x^2)^2 - (x^2)^2]$$

maximize

$$A(x) = \pi (36x^2 - 12x^3 + \cancel{x^4} - \cancel{x^4})$$

$$A = 12x^2\pi(3 - x)$$

$$A' =$$

$$A' = \pi(72x - 36x^2)$$

$$36x\pi(2 - x) \stackrel{\text{Set}}{=} 0$$

$$x = 2$$

0	+	0	-	max $f'(x)$
1		2		

22. $\int \frac{e^u}{4 + e^{2u}} du$ is equal to

(A) $\ln(4 + e^{2u}) + C$ (B) $\frac{1}{2} \ln|4 + e^{2u}| + C$

(C) $\frac{1}{2} \tan^{-1} \frac{e^u}{2} + C$ (D) $\tan^{-1} \frac{e^u}{2} + C$

(E) $\frac{1}{2} \tan^{-1} \frac{e^{2u}}{2} + C$

*23. $\int_2^4 \frac{du}{\sqrt{16 - u^2}}$ equals

(A) $\frac{\pi}{12}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{3}$ (E) $\frac{2\pi}{3}$

24. $\int \frac{dx}{x \ln x}$ equals

(A) $\ln(\ln x) + C$ (B) $-\frac{1}{\ln^2 x} + C$ (C) $\frac{(\ln x)^2}{2} + C$

(D) $\ln x + C$ (E) none of these.

25. If we replace $\sqrt{x-2}$ by u then $\int_3^6 \frac{\sqrt{x-2}}{x} dx$ is equivalent to

(A) $\int_1^2 \frac{u du}{u^2 + 2}$ (B) $2 \int_1^2 \frac{u^2 du}{u^2 + 2}$ (C) $\int_3^6 \frac{2u^2}{u^2 + 2} du$

(D) $\int_3^6 \frac{u du}{u^2 + 2}$ (E) $\frac{1}{2} \int_1^2 \frac{u^2}{u^2 + 2} du$

*26. $\int_2^4 \frac{du}{(u-3)^2}$ equals

(A) 2 (B) 1 (C) -1 (D) -2 (E) none of these.

*27. Which of the following improper integrals diverges?

(A) $\int_0^\infty e^{-x^2} dx$

(B) $\int_{-\infty}^0 e^x dx$

(C) $\int_0^1 \frac{dx}{x}$

(D) $\int_0^\infty e^{-x} dx$

(E) $\int_0^1 \frac{dx}{\sqrt{x}}$

28. If $\frac{dQ}{dt} = \frac{Q}{10}$ and $Q = Q_0$ when $t = 0$, then Q , at any time t , equals

(A) $Q_0 \cdot 10^t$ (B) $Q_0 \cdot e^{10t}$ (C) $Q_0 \cdot t^{1/10}$
 (D) $Q_0 \cdot e^{t/10}$ (E) $\frac{10Q_0}{10-t}$

29. If a function $f(x)$ is defined by $f(x) = 2x$ for $x \leq 1$,
 $f(x) = 3x^2 - 2$ for $x > 1$,

then $\int_0^2 f(x) dx$ equals $\int_0^1 2x dx + \int_1^2 (3x^2 - 2) dx = x^2 \Big|_0^1 + (x^3 - 2x) \Big|_1^2$
 $= 1 + (8 - 4 - 1 + 2) = 6$
 (A) 6 (B) 5 (C) 4 (D) 3 (E) none of these.

30. The area bounded by the parabola $y = x^2$ and the lines $y = 1$

and $y = 9$ equals

(A) 8 (B) $\frac{84}{3}$ (C) $\frac{64}{3}\sqrt{2}$ (D) 32 (E) $\frac{104}{3}$

31. The total area bounded by the curves $y = x^3$ and $y = x^{1/3}$

is equal to

(A) $\frac{1}{2}$ (B) $\frac{5}{6}$ (C) 1 (D) 2 (E) none of these.

32. The area under the curve $y = \frac{1}{\sqrt{x-1}}$, above the x -axis and bounded

vertically by $x = 2$ and $x = 5$ equals

(A) 1 (B) 2 (C) 4 (D) $\ln 2$ (E) $2 \ln 2$

33. The total area enclosed by the curves $y = 4x - x^3$ and $y = 4 - x^2$

is represented by

(A) $\int_{-2}^2 (4x - x^3 + x^2 - 4) dx$ (B) $\int_{-2}^2 (4 - x^2 - 4x + x^3) dx$
 (C) $\int_{-2}^1 (4x - x^3 + x^2 - 4) dx + \int_1^2 (4 - x^2 - 4x + x^3) dx$
 (D) $\int_{-2}^1 (4 - x^2 - 4x + x^3) dx + \int_1^2 (4x - x^3 + x^2 - 4) dx$
 (E) $\int_{-2}^1 (4x - x^3 + x^2 - 4) dx$

- *34. The region bounded by $y = \tan x$, $y = 0$, and $x = \frac{\pi}{4}$ is rotated about the x -axis. The volume generated equals

(A) $\pi - \frac{\pi^2}{4}$ (B) $\pi(\sqrt{2} - 1)$ (C) $\frac{3\pi}{4}$ (D) $\pi(1 + \frac{\pi}{4})$
(E) none of these.

- *35. The volume obtained by rotating the region bounded by $x = y^2$ and $x = 2 - y^2$ about the y -axis is equal to

(A) $\frac{16\pi}{3}$ (B) $\frac{32\pi}{3}$ (C) $\frac{32\pi}{15}$ (D) $\frac{64\pi}{15}$ (E) $\frac{8\pi}{3}$.

- *36. The first-quadrant region bounded by $y = \frac{1}{\sqrt{x}}$, $y = 0$, $x = q$ ($0 < q < 1$), and $x = 1$ is rotated about the x -axis. The volume obtained as $q \rightarrow 0^+$ equals

(A) $\frac{2\pi}{3}$ (B) $\frac{4\pi}{3}$ (C) 2π (D) 4π (E) none of these.

- *37. A curve is given parametrically by the equations $x = 3 - 2 \sin t$ and $y = 2 \cos t - 1$. The length of the arc from $t = 0$ to $t = \pi$ is

(A) $\frac{\pi}{2}$ (B) π (C) $2 + \pi$ (D) 2π (E) 4π .

- *38. The area in square inches of the surface of a zone cut from a sphere of radius 4 inches by two parallel planes, one through the center of the sphere and the other one inch away, is equal to

(A) 2π (B) 4π (C) 8π (D) 16π (E) 20π .

39. The average (mean) value of $y = \ln x$ over the interval from $x = 1$ to $x = e$ equals

(A) $\frac{1}{2}$ (B) $\frac{1}{2(e-1)}$ (C) $\frac{2}{e-1}$ (D) $\frac{1}{e-1}$

(E) none of these.

40. A particle moves along a line with acceleration $a = 6t$. If, when $t = 0$, $v = 1$, then the total distance travelled between $t = 0$ and $t = 3$ equals

(A) 30 (B) 28 (C) 27 (D) 26 (E) none of these.

41. $\lim_{n \rightarrow \infty} \frac{1}{n} (\cos \frac{2}{n} + \cos \frac{4}{n} + \cos \frac{6}{n} + \dots + \cos \frac{2n}{n})$ equals

(A) $\sin 2$ (B) $\frac{1}{2} \sin 2$ (C) $-\sin 2$ (D) $\frac{1}{2} (\sin 2 - \sin 1)$

(E) none of these.

42. If $x = f(t)$ is the law of motion for a particle on a line, the particle is said to be in simple harmonic motion if the acceleration

$\frac{d^2x}{dt^2} = -k^2x$, k a constant. Which of the following equations does

not define a simple harmonic motion?

(A) $x = e^t + e^{-t}$

(B) $x = 4 \cos 2t$

(C) $x = 2 \sin (\pi t + 2)$

(D) $x = \sin 2t + \cos 2t$

(E) $x = \cos (\omega t + \phi)$, ω and ϕ constants.

43. The graph of the equation $y^2 = 4(x+1)^2 + 4$ is

(A) a parabola

(B) an ellipse

(C) two intersecting

straight lines

(D) a circle

(E) a hyperbola.

44. The graph of $y = \sin \frac{1}{x}$ is symmetric to

(A) the x -axis

(B) the y -axis

(C) the line $y = x$

(D) the origin

(E) none of these.

45. Which of the following functions has a graph which is asymptotic to the y -axis?

(A) $y = \frac{x}{x^2 - 1}$

(B) $y = \frac{x}{x^2 + 1}$

(C) $y = x - \frac{2}{x}$

(D) $y = \ln (x - 1)$

(E) $y = \frac{\sqrt{x-1}}{x}$

A.P. Multiple Choice # 5

Mr. Mumaugh

Set a clock. Work in a room with as few distractions as possible. You need to simulate test conditions.

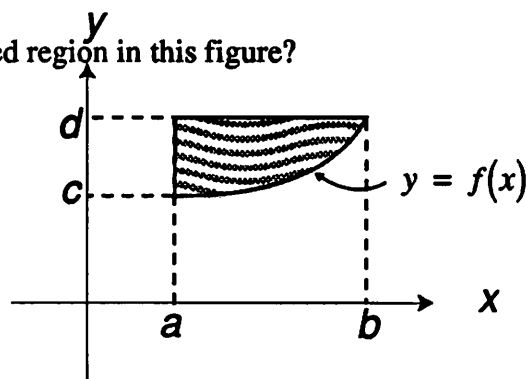
Note: The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value. Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. If $f(x) = x^{\frac{3}{2}}$, then $f'(4) =$

- a) -6 b) -3 c) 3 d) 6 e) 8

2. Which of the following represents the area of the shaded region in this figure?

- a) $\int_c^d f(y) dy$ b) $\int_a^b (d - f(x)) dx$
c) $f'(b) - f'(a)$ d) $(b - a)[f'(b) - f'(a)]$
e) $(d - c)[f(b) - f(a)]$



3. $\lim_{n \rightarrow \infty} \frac{3n^3 - 5n}{n^3 - 2n^2 + 1} =$

- a) -5 b) -2 c) 1 d) 3 e) nonexistent

4. If $x^3 + 3xy + 2y^3 = 17$, then in terms of x and y , $\frac{dy}{dx} =$

- a) $-\frac{x^2 + y}{x + 2y^2}$ b) $-\frac{x^2 + y}{x + y^2}$ c) $-\frac{x^2 + y}{x + 2y}$ d) $-\frac{x^2 + y}{2y^2}$ e) $\frac{-x^2}{1 + 2y^2}$

5. If the function f is continuous for all real numbers and if $f(x) = \frac{x^2 - 4}{x + 2}$ when $x \neq -2$, then $f(-2) =$

- a) -4 b) -2 c) -1 d) 0 e) 2

6. The area of the region enclosed by the curve $y = \frac{1}{x-1}$, the x -axis, and the lines $x=3$ and $x=4$ is

- a) $\frac{5}{36}$ b) $\ln \frac{2}{3}$ c) $\ln \frac{4}{3}$ d) $\ln \frac{3}{2}$ e) $\ln 6$

A.P. Multiple Choice # 5

- ☾ An equation of the line tangent to the graph of $y = \frac{2x + 3}{3x - 2}$ at the point $(1, 5)$ is
- a) $13x - y = 8$ b) $13x + y = 18$ c) $x - 13y = 64$ d) $x + 13y = 66$ e) $3y - 2x = 13$
- 8) $y = \tan x - \cot x$, then $\frac{dy}{dx} =$
- a) $\sec x \csc x$ b) $\sec x - \csc x$ c) $\sec x + \csc x$ d) $\sec^2 x - \csc^2 x$ e) $\sec^2 x + \csc^2 x$
- 9) If h is the function given by $h(x) = f(g(x))$, where $f(x) = 3x^2 - 1$ and $g(x) = |x|$, then $h(x) =$
- a) $3x^3 - |x|$ b) $|3x^2 - 1|$ c) $3x^2|x| - 1$ d) $3|x| - 1$ e) $3x^2 - 1$
- 10) If $f(x) = (x - 1)^2 \sin x$, then $f'(0) =$
- a) -2 b) -1 c) 0 d) 1 e) 2
- 11) The acceleration of a particle moving along the x -axis at time t is given by $a(t) = 6t - 2$.
- ☾ If the velocity is 25 when $t = 3$ and the position is 10 when $t = 1$, then the position $x(t) =$
- a) $9t^2 + 1$ b) $3t^2 - 2t + 4$ c) $t^3 - t^2 + 4t + 6$
- d) $t^3 - t^2 + 9t - 20$ e) $36t^3 - 4t^2 - 77t + 55$
- 12) If f and g are continuous functions, and if $f(x) \leq 0$ all real numbers x , which of the following must be true?
- I. $\int_a^b f(x)g(x) \, dx = \left(\int_a^b f(x) \, dx \right) \left(\int_a^b g(x) \, dx \right)$
- II. $\int_a^b (f(x) + g(x)) \, dx = \int_a^b f(x) \, dx + \int_a^b g(x) \, dx$
- III. $\int_a^b \sqrt{f(x)} \, dx = \sqrt{\int_a^b f(x) \, dx}$
- a) I only b) II only c) III only d) II and III only e) I, II, and III
- ☾

A.P. Multiple Choice # 5

13) The fundamental period of $2 \cos(3x)$ is

- a) $\frac{2\pi}{3}$ b) 2π c) 6π d) 2 e) 3

14) $\int \frac{3x^2}{\sqrt{x^3 + 1}} dx =$

- a) $2\sqrt{x^3 + 1} + C$ b) $\frac{3}{2}\sqrt{x^3 + 1} + C$ c) $\sqrt{x^3 + 1} + C$ d) $\ln \sqrt{x^3 + 1} + C$ e) $\ln(x^3 + 1) + C$

15) For what value of x does the function $f(x) = (x - 2)(x - 3)^2$ have a relative maximum?

- a) -3 b) $-\frac{7}{3}$ c) $-\frac{5}{2}$ d) $\frac{7}{3}$ e) $\frac{5}{2}$

16) The slope of the line normal to the graph of $y = 2 \ln(\sec x)$ at $x = \frac{\pi}{4}$ is

- a) -2 b) $-\frac{1}{2}$ c) $\frac{1}{2}$ d) 2 e) nonexistent

17) $\int (x^2 + 1)^2 dx =$

- a) $\frac{(x^2 + 1)^3}{3} + C$ b) $\frac{(x^2 + 1)^3}{6x} + C$ c) $\left(\frac{x^3}{3} + x\right)^2 + C$
d) $\frac{2x(x^2 + 1)^3}{3} + C$ e) $\frac{x^5}{5} + \frac{2x^3}{3} + x + C$

18) If $f(x) = \sin\left(\frac{x}{2}\right)$, then there exists a number c in the interval $\frac{\pi}{2} < x < \frac{3\pi}{2}$ that satisfies the conclusion of the Mean Value Theorem. Which of the following could be c ?

- a) $\frac{2\pi}{3}$ b) $\frac{3\pi}{4}$ c) $\frac{5\pi}{6}$ d) π e) $\frac{3\pi}{2}$

A.P. Multiple Choice # 5

- 19) Let f be the function defined by $f(x) = \begin{cases} x^3 & \text{for } x \leq 0 \\ x & \text{for } x > 0 \end{cases}$.

Which of the following statements about f is true?

- a) f is an odd function b) f is discontinuous at $x = 0$ c) f has a relative maximum
d) $f'(0) = 0$ e) $f'(x) > 0$ for $x \neq 0$

- 20) Let R be the region in the first quadrant enclosed by the graph of $y = (x + 1)^{\frac{1}{3}}$, the line $x = 7$, the x -axis and the y -axis. The volume of the solid generated when R is revolved about the y -axis would be given by

- a) $\pi \int_0^7 (x + 1)^{\frac{2}{3}} dx$ b) $2\pi \int_0^7 x(x + 1)^{\frac{1}{3}} dx$ c) $\pi \int_0^2 (x + 1)^{\frac{2}{3}} dx$
d) $2\pi \int_0^2 x(x + 1)^{\frac{1}{3}} dx$ e) $\pi \int_0^7 (y^3 - 1)^2 dy$

- 21) At what value of x does the graph of $y = \frac{1}{x^2} - \frac{1}{x^3}$ have a point of inflection?

- a) 0 b) 1 c) 2 d) 3 e) At no value of x

- 22) An antiderivative for $\frac{1}{x^2 - 2x + 2}$ is

- a) $-(x^2 - 2x + 2)^{-2}$ b) $\ln(x^2 - 2x + 2)$ c) $\ln \left| \frac{x - 2}{x + 1} \right|$
d) $\text{Arcsec}(x - 1)$ e) $\text{Arctan}(x - 1)$

- 23) How many critical points does the function $f(x) = (x + 2)^5(x - 3)^4$ have?

- a) one b) two c) three d) five e) nine

- 24) If $f(x) = (x^2 - 2x - 1)^{\frac{2}{3}}$, then $f'(0)$ is

- a) $\frac{4}{3}$ b) 0 c) $-\frac{2}{3}$ d) $-\frac{4}{3}$ e) -2

A.P. Multiple Choice # 5

$\frac{d}{dx}(2^x) =$

- a) 2^{x-1} b) $(2^{x-1})x$ c) $(2^x) \ln 2$ d) $(2^{x-1}) \ln 2$ e) $\frac{2x}{\ln 2}$

26) A particle moves along a line so that at time t , where $0 \leq t \leq \pi$, its position is given by $s(t) = -4 \cos t - \frac{t^2}{2} + 10$. What is the velocity of the particle when its acceleration is zero?

- a) -5.19 b) 0.74 c) 1.32 d) 2.55 e) 8.13

27) The function f given by $f(x) = x^3 + 12x - 24$ is

- a) increasing for $x < -2$, decreasing for $-2 < x < 2$, increasing for $x > 2$
 b) decreasing for $x < 0$, increasing for $x > 0$
 c) increasing for all x
 d) decreasing for all x
 e) decreasing for $x < -2$, increasing for $-2 < x < 2$, decreasing for $x > 2$

28) $\int_1^{500} (13^x - 11^x) dx + \int_2^{500} (11^x - 13^x) dx =$

- a) 0.000 b) 14.946 c) 34.415 d) 46.000 e) 136.364

29) $\lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{2 \sin^2 \theta}$ is

- a) 0 b) $\frac{1}{8}$ c) $\frac{1}{4}$ d) 1 e) nonexistent

30) The region enclosed by the x -axis, the line $x = 3$, and the curve $y = \sqrt{x}$ is rotated about the x -axis. What is the volume of the solid generated?

- a) 3π b) $2\sqrt{3}\pi$ c) $\frac{9}{2}\pi$ d) 9π e) $\frac{36\sqrt{3}}{5}\pi$

A.P. Multiple Choice # 5

☾ If $f(x) = e^{3\ln(x^2)}$, then $f'(x) =$

- a) $e^{3\ln(x^2)}$ b) $\frac{3}{x^2} e^{3\ln(x^2)}$ c) $6(\ln x)e^{3\ln(x^2)}$ d) $5x^4$ e) $6x^5$

32) $\int_0^{\sqrt{3}} \frac{dx}{\sqrt{4-x^2}} =$

- a) $\frac{\pi}{3}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{6}$ d) $\frac{1}{2} \ln 2$ e) $-\ln 2$

33) $\frac{dy}{dx} = 2y^2$ and if $y = -1$ when $x = 1$, then when $x = 2$, $y =$

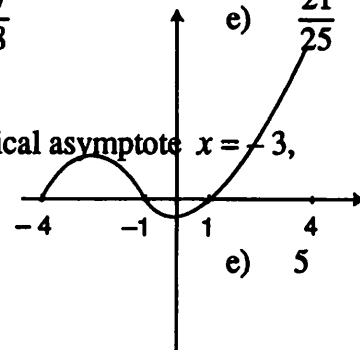
- a) $-\frac{2}{3}$ b) $-\frac{1}{3}$ c) 0 d) $\frac{1}{3}$ e) $\frac{2}{3}$

34) The top of a 25-foot ladder is sliding down a vertical wall at a constant rate of 3 feet per minute. When the top of the ladder is 7 feet from the ground, what is the rate of change, in feet per minute, of the distance between the bottom of the ladder and the wall?

- ☾ a) $-\frac{7}{8}$ b) $-\frac{7}{24}$ c) $\frac{7}{24}$ d) $\frac{7}{8}$ e) $\frac{21}{25}$

35) If the graph of $y = \frac{ax+b}{x+c}$ has a horizontal asymptote $y = 2$ and a vertical asymptote $x = -3$, then $a + c =$

- a) -5 b) -1 c) 0 d) 1 e) 5



36) If the definite integral $\int_0^2 e^{x^2} dx$ is first approximated by using two inscribed rectangles of equal width and then approximated by using the trapezoidal rule with $n = 2$, the difference between the two approximations is

- a) 53.60 b) 30.51 c) 27.80 d) 26.80 e) 12.78

A.P. Multiple Choice # 5

If f is a differentiable function, then $f'(a)$ is given by which of the following?

- I. $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$
 II. $\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$
 III. $\lim_{x \rightarrow a} \frac{f(x+h) - f(x)}{h}$

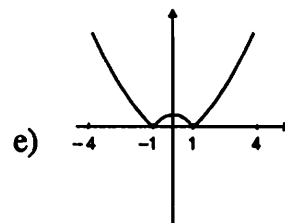
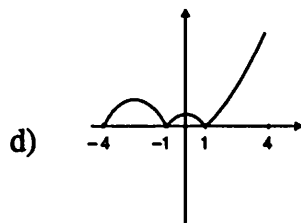
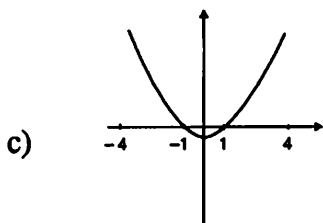
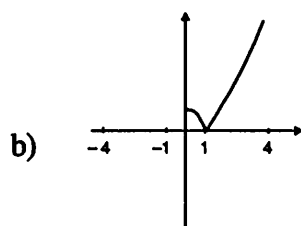
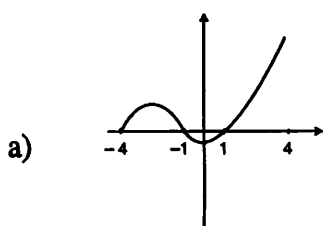
- a) I only b) II only c) I and II only d) I and III only e) I, II, and III

38) If the second derivative of f is given by $f''(x) = 2x - \cos x$, which of the following could be $f(x)$?

- a) $\frac{x^3}{3} + \cos x - x + 1$ b) $\frac{x^3}{3} - \cos x - x + 1$ c) $x^3 + \cos x - x + 1$
 d) $x^2 - \sin x - x + 1$ e) $x^2 + \sin x - x + 1$

39) The graph of $y = f(x)$ is shown in the figure to the right.

Which of the following could be the graph of $y = f(|x|)$



40) The radius of a circle is increasing at a nonzero rate, and at a certain instant, the rate of increase in the area of the circle is numerically equal to the rate of increase in its circumference. At this instant, the radius of the circle is

- a) $\frac{1}{\pi}$ b) $\frac{1}{2}$ c) $\frac{2}{\pi}$ d) 1 e) 2

A.P. Multiple Choice # 5

41) $\frac{d}{dx} \int_0^x \cos(2\pi u) du$

- a) 0 b) $\frac{1}{2\pi} \sin x$ c) $\frac{1}{2\pi} \cos(2\pi x)$ d) $\cos(2\pi x)$ e) $2\pi \cos(2\pi x)$

42) A puppy weighs 2.0 pounds at birth and 3.5 pounds two months later. If the weight of the puppy during its first 6 months is increasing at a rate proportional to its weight, then how much (in pounds) will the puppy weigh when it is 3 months old?

- a) 4.2 b) 4.6 c) 4.8 d) 5.6 e) 6.5

43) $\int x f(x) dx =$

- a) $x f(x) - \int x f'(x) dx$ b) $\frac{x^2}{2} f(x) - \int \frac{x^2}{2} f'(x) dx$ c) $x f(x) - \frac{x^2}{2} f'(x) + C$
d) $x f(x) - \int f'(x) dx$ e) $\int \frac{x^2}{2} f(x) dx$

44) What is the minimum value of $f(x) = x \ln x$?

- a) $-e$ b) -1 c) $-\frac{1}{e}$ d) 0 e) $f(x)$ has no minimum value

45) If Newton's method is used to approximate the real root of $x^3 + x - 1 = 0$, then a first approximation $x_1 = 1$ would lead to a third approximation of $x_3 =$

- a) 0.682 b) 0.686 c) 0.694 d) 0.750 e) 1.637

46) The area of the region enclosed by the graphs of $y = x^2$ and $y = x$ is

- a) $\frac{1}{6}$ b) $\frac{1}{3}$ c) $\frac{1}{2}$ d) $\frac{5}{6}$ e) 1

47) If $f(x) = 2x^2 + 1$, then $\lim_{x \rightarrow 0} \frac{f(x) - f(0)}{x^2}$ is

- a) 0 b) 1 c) 2 d) 4 e) nonexistent

48) If p is a polynomial of degree n , $n > 0$, what is the degree of the polynomial $Q(x) = \int_0^x p(t) dt$?

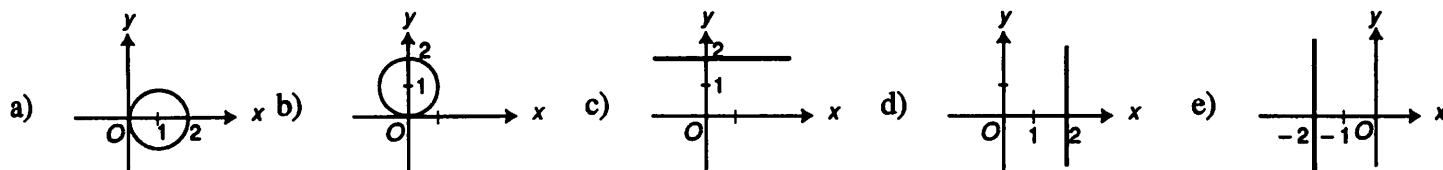
- a) 0 b) 1 c) $n - 1$ d) n e) $n + 1$

A.P. Multiple Choice # 5

 A particle moves along the curve $xy = 10$. If $x = 2$ and $\frac{dy}{dt} = 3$, what is the value of $\frac{dx}{dt}$?

- a) $-\frac{5}{2}$ b) $-\frac{6}{5}$ c) 0 d) $\frac{4}{5}$ e) $\frac{6}{5}$

50) Which of the following represents the graph of the polar curve $r = 2 \sec \theta$?



51) If $x = t^2 + 1$ and $y = t^3$, then $\frac{d^2y}{dx^2} =$

- a) $\frac{3}{4t}$ b) $\frac{3}{2t}$ c) $3t$ d) $6t$ e) $\frac{3}{2}$

 52) $\int_0^1 x^3 e^{x^4} dx =$

- a) $\frac{1}{4}(e - 1)$ b) $\frac{1}{4}e$ c) $e - 1$ d) e e) $4(e - 1)$

53) If $f(x) = \ln(e^{2x})$, then $f'(x) =$

- a) 1 b) 2 c) $2x$ d) e^{-2x} e) $2e^{-2x}$

54) If $f(x) = 1 + x^{\frac{2}{3}}$, which of the following is NOT true?

- a) f is continuous for all real numbers. b) f has a minimum at $x = 0$.
c) f is increasing for $x > 0$. d) $f'(x)$ exists for all x .
e) $f''(x)$ is negative for $x > 0$.

55) Which of the following functions are continuous at $x = 1$?

I. $\ln x$

II. e^x

III. $\ln(e^x - 1)$

-  a) I only b) II only c) I and II only d) II and III only e) I, II, and III

A.P. Multiple Choice # 5

57) $\int_4^{\infty} \frac{-2x}{\sqrt[3]{9-x^2}} dx$ is

- a) $7^{\frac{2}{3}}$ b) $\frac{3}{2} \left(7^{\frac{2}{3}} \right)$ c) $9^{\frac{2}{3}} + 7^{\frac{2}{3}}$ d) $\frac{3}{2} \left(9^{\frac{2}{3}} + 7^{\frac{2}{3}} \right)$ e) nonexistent

58) The position of a particle moving along the x -axis is $x(t) = \sin(2t) - \cos(3t)$ for time $t \geq 0$. When $t = \pi$, the acceleration of the particle is

- a) 9 b) $\frac{1}{9}$ c) 0 d) $-\frac{1}{9}$ e) -9

59) If $\frac{dy}{dx} = x^2 y$, then y could be

- a) $3 \ln\left(\frac{x}{3}\right)$ b) $e^{\frac{x^3}{3}} + 7$ c) $2e^{\frac{x^3}{3}}$ d) $3e^{2x}$ e) $\frac{x^3}{3} + 1$

60) The derivative of f is $x^4(x-2)(x+3)$. At how many points will the graph of f have a relative maximum?

- a) None b) One c) Two d) Three e) Four

61) If $f(x) = e^{\tan^2 x}$, then $f'(x) =$

- a) $e^{\tan^2 x}$ b) $\sec^2 x e^{\tan^2 x}$ c) $\tan^2 x e^{\tan^2 x - 1}$ d) $2 \tan x \sec^2 x e^{\tan^2 x}$ e) $2 \tan x e^{\tan^2 x}$

62) If f is continuous on the closed interval $[a, b]$, then there exists c such that $a < c < b$ and $\int_a^b f(x) dx =$

- a) $\frac{f(c)}{b-a}$ b) $\frac{f(b) - f(a)}{b-a}$ c) $f(b) - f(a)$ d) $f'(c)(b-a)$ e) $f(c)(b-a)$

63) The slope of the line tangent to the graph of $\ln(xy) = x$ at the point where $x = 1$ is

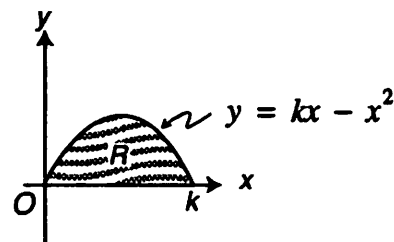
- a) 0 b) 1 c) e d) e^2 e) $1-e$

64) If $e^{f(x)} = 1 + x^2$, then $f'(x) =$

- a) $\frac{1}{1+x^2}$ b) $\frac{2x}{1+x^2}$ c) $2x(1+x^2)$ d) $2x(e^{1+x^2})$ e) $2x \ln(1+x^2)$

A.P. Multiple Choice # 5

- 64) The shaded region shown on the right is rotated about the y-axis to form a solid whose volume is 10 cubic units. Of the following, which best approximates k ?



- a) 1.51 b) 2.09 c) 2.49 d) 4.18 4.77
- 65) A particle moves along the x -axis so that at any time $t \geq 0$ the acceleration of the particle is $a(t) = e^{-2t}$. If at $t = 0$ the velocity of the particle is $\frac{5}{2}$ and its position is $\frac{17}{4}$, then its position at any time $t \geq 0$ is
- a) $x(t) = -\frac{e^{2t}}{2} + 3$ b) $x(t) = \frac{e^{-2t}}{4} + 4$ c) $x(t) = 4e^{-2t} + \frac{9}{2}t + \frac{1}{4}$
- d) $x(t) = \frac{e^{-2t}}{2} + 3t + \frac{15}{4}$ e) $x(t) = \frac{e^{-2t}}{4} + 3t + 4$

- The value of the derivative of $y = \frac{\sqrt[3]{x^2 + 8}}{\sqrt{2x + 1}}$ at $x = 0$ is

- a) -1 b) $-\frac{1}{2}$ c) 0 d) $\frac{1}{2}$ e) 1
- 67) If $f(x) = x^2 e^x$, then the graph of f is decreasing for all x such that
- a) $x < -2$ b) $-2 < x < 0$ c) $x > -2$ d) $x < 0$ e) $x > 0$
- 68) The length of the curve determined by the equations $x = t^2$ and $y = t$ from $t = 0$ to $t = 4$ is
- a) $\int_0^4 \sqrt{4t + 1} \, dt$ b) $2 \int_0^4 \sqrt{t^2 + 1} \, dt$ c) $\int_0^4 \sqrt{2t^2 + 1} \, dt$
- d) $\int_0^4 \sqrt{4t^2 + 1} \, dt$ e) $2\pi \int_0^4 \sqrt{4t^2 + 1} \, dt$

A.P. Multiple Choice # 5

69) Let f and g be functions that are differentiable for all real numbers,

If $\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} g(x) = 0$ and $\lim_{x \rightarrow 0} \frac{f'(x)}{g'(x)}$ exists, then $\lim_{x \rightarrow 0} \frac{f(x)}{g(x)}$ is

- a) 0 b) $\frac{f'(x)}{g'(x)}$ c) $\lim_{x \rightarrow 0} \frac{f'(x)}{g'(x)}$ d) $\frac{f'(x)g(x) - f(x)g'(x)}{(f(x))^2}$ e) nonexistent

70) Consider the curve in the xy -plane represented by $x = e^t$ and $y = te^{-t}$ for $t \geq 0$. The slope of the line tangent to the curve at the point where $x = 3$ is

- a) 20.086 b) 0.342 c) -0.005 d) -0.011 e) -0.033

71) If $y = \text{Arctan}(e^{2x})$, then $\frac{dy}{dx} =$

- a) $\frac{2e^{2x}}{\sqrt{10 - e^{4x}}}$ b) $\frac{2e^{2x}}{1 + e^{4x}}$ c) $\frac{e^{2x}}{1 + e^{4x}}$ d) $\frac{1}{\sqrt{1 - e^{4x}}}$ e) $\frac{1}{1 + e^{4x}}$

$\int x \sec^2 x \, dx =$

- a) $x \tan x + C$ b) $\frac{x^2}{2} \tan x + C$ c) $\sec^2 x + 2 \sec^2 x \tan x + C$
 d) $x \tan x - \ln|\cos x| + C$ e) $x \tan x + \ln|\cos x| + C$

73) What is the volume of the solid generated by rotating about the x -axis the region enclosed by the curve $y = \sec x$ and the lines $x = 0$, $y = 0$, and $x = \frac{\pi}{3}$?

- a) $\frac{\pi}{\sqrt{3}}$ b) π c) $\pi\sqrt{3}$ d) $\frac{8\pi}{3}$ e) $\pi \ln\left(\frac{1}{2} + \sqrt{3}\right)$

A.P. Multiple Choice # 5

If $\int_a^b f(x) \, dx = 5$ and $\int_a^b g(x) \, dx = -1$, which of the following must be true?

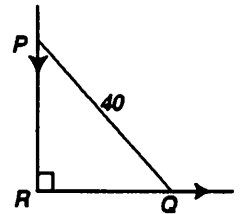
- I. $f(x) > g(x)$ for $a \leq x \leq b$
- II. $\int_a^b [f(x) + g(x)] \, dx = 4$
- III. $\int_a^b [f(x)g(x)] \, dx = -5$

- a) I only b) II only c) III only d) II and III only e) I, II, and III

75) Which of the following is equal to $\int_0^\pi \sin x \, dx$?

- a) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \, dx$ b) $\int_0^\pi \cos x \, dx$ c) $\int_{-\pi}^0 \sin x \, dx$ d) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin x \, dx$ e) $\int_0^{2\pi} \sin x \, dx$

76) In the figure on the right, PQ represents a 40-foot ladder with end P against a vertical wall and end Q on level ground. If the ladder is slipping down the wall, what is the distance RQ at the instant when Q is moving along the ground $\frac{3}{4}$ as fast as P is moving down the wall?



- a) $\frac{6}{5}\sqrt{10}$ b) $\frac{8}{5}\sqrt{10}$ c) $\frac{80}{\sqrt{7}}$ d) 24 e) 32

77) If F and f are differentiable functions such that $F(x) = \int_0^x f(t) \, dt$, and if $F(a) = -2$ and $F(b) = -2$ where $a < b$, which of the following must be true?

- a) $f(x) = 0$ for some x such that $a < x < b$.
- b) $f(x) > 0$ for all x such that $a < x < b$.
- c) $f(x) < 0$ for all x such that $a < x < b$.
- d) $F(x) \leq 0$ for all x such that $a < x < b$.
- e) $F(x) = 0$ for some x such that $a < x < b$.

A.P. Multiple Choice # 5

78) Consider all right circular cylinders for which the sum of the height and the circumference is 30 centimeters. What is the radius of the one with maximum volume?

- a) 3 cm b) 10 cm c) 20 cm d) $\frac{30}{\pi^2}$ cm e) $\frac{10}{\pi}$ cm

79) If $f(x) = \begin{cases} x & \text{for } x \leq 1 \\ \frac{1}{x} & \text{for } x > 1 \end{cases}$, then $\int_0^e f(x) dx =$

- a) 0 b) $\frac{3}{2}$ c) 2 d) e e) $e + \frac{1}{2}$

80) During a certain epidemic, the number of people that are infected at any time increases at a rate proportional to the number of people that are infected at that time. If 1,000 people are infected when the epidemic is first discovered, and 1,200 are infected 7 days later, how many people are infected 12 days after the epidemic is first discovered?

- a) 343 b) 1,343 c) 1,367 d) 1,400 e) 2,057

81) If $\frac{dy}{dx} = \frac{1}{x}$, then the average rate of change of y with respect to x on the closed interval $[1, 4]$ is

- a) $-\frac{1}{4}$ b) $\frac{1}{2} \ln 2$ c) $\frac{2}{3} \ln 2$ d) $\frac{2}{5}$ e) 2

82) Let R be the region in the first quadrant enclosed by the x -axis and the graph of $y = \ln(1 + 2x - x^2)$. If Simpson's Rule with 2 subintervals is used to approximate the area of R , the approximation is

- a) 10.462 b) 0.693 c) 0.924 d) 0.986 e) 1.850

83) Let $f(x) = \int_{-2}^{x^2-3x} e^{t^2} dt$. At what value of x is $f(x)$ a minimum?

- a) For no value of x b) $\frac{1}{2}$ c) $\frac{3}{2}$ d) 2 e) 3

84) $\lim_{x \rightarrow 0} (1 + 2x)^{\csc x} =$

- a) 0 b) 1 c) 2 d) e e) e^2

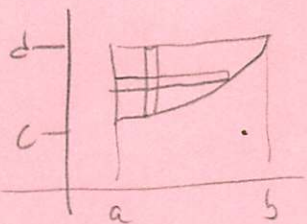
MC # 5 put on boards.

(11) C	(39) C	(65) E
(18) D	(42) B	(66) A
(19) E	(44) C	(70) D
(21) C	(45) B	(76) E
(23) C	(51) A	(77) A
(28) B	(54) B	(78) E
(32) A	(56) E	(82) C
(33) B	(58) C	(83) C
(35) E	(62) A	(84) E
(36) D	(64) B	

People	Planes
2	11, 18, 19
2	21, 23, 28
2	32, 33, 35
2	36, 39, 42
2	44, 45, 51
	54, 56
	58, 62
	64, 65
	66, 70

76, 77
78, 82
83, 84

1. $f(x) = x^{\frac{3}{2}}$
 $f'(x) = \frac{3}{2} x^{\frac{1}{2}} \Big|_{x=4} = \frac{3}{2} \cdot 2 = 3 \quad (C)$

2.  $\int_a^b d - f(x) dx \quad (B)$
 $\int_c^d (y - a) dy$ via change.

3. $\lim_{N \rightarrow \infty} \frac{3N^3 - 5N}{N^3 - 2N^2 + 1} \cdot \frac{\frac{1}{N^3}}{\frac{1}{N^3}} = 3 \quad (D)$

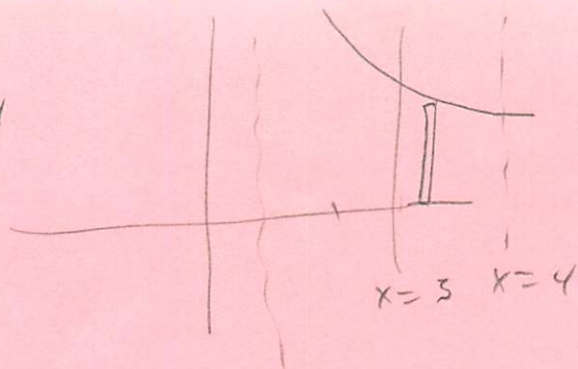
4. $x^3 + 3xy + 2y^3 = 17$
 $\frac{d}{dx} (x^3 + 3xy + 2y^3) = 0$
 $3x^2 + 3y + 3x \frac{dy}{dx} + 6y^2 \frac{dy}{dx} = 0$
 $\frac{dy}{dx} = \frac{-(x^2 + y)}{(x + 2y^2)} \quad (A)$

5. $f(x) = \frac{x^2 - 4}{x + 2}$
 $\frac{(x+2)(x-2)}{(x+2)}$

$f(x) = x - 2$
 f is continuous $\therefore f(-2) = -2 - 2 = -4 \quad (A)$

6.

$$y = \frac{1}{x-1}$$



$$\int_3^4 \frac{1}{x-1} dx$$

$$\ln|x-1| \Big|_3^4$$

(D)

$$\ln 3 - \ln 2 = \ln \frac{3}{2}$$

7.

$$y = \frac{2x+3}{3x-2}$$

$$y' = \frac{2(3x-2) - 3(2x+3)}{(3x-2)^2}$$

$$\frac{6x - 4 - 6x - 9}{(3x-2)^2}$$

$$\left. \frac{-13}{(3x-2)^2} \right|_{x=1} = -13 = m$$

$$y - 5 = -13(x - 1)$$

$$y = -13x + 18$$

(B)

8.

$$y = \tan x - \cot x$$

$$y' = \sec^2 x + \csc^2 x \quad (E)$$

9.

$$h = f(g(x))$$

$$f(x) = 3x^2 - 1$$

$$g(x) = |x|$$

$$h = f(|x|) = 3|x|^2 - 1 \quad (E)$$

10.

$$f(x) = (x-1)^2 \sin x$$

$$f'(x) = 2(x-1) \sin x + (x-1)^2 \cos x$$

$$f'(0) = -2(0) + 1 \cdot 1 = 1 \quad (D)$$

11.

$$a = 6t - 2$$

$$v(3) = 25$$

$$x(1) = 10$$

$$v = \int (6t - 2) dt$$

$$3t^2 - 2t + C \Big|_3^{3t} = 25$$

$$27 - 6 + C = 25$$

$$C = 4$$

$$x(t) = t^3 - t^2 + 4t + 6 \quad \textcircled{C}$$

$$CK \begin{cases} 3t^2 - 2t + 4 \Big|_3 = \\ 6t - 2 \end{cases}$$

$$x = \int (3t^2 - 2t + 4) dt$$

$$t^3 - t^2 + 4t + C \Big|_1^{3t} = 10$$

$$1 - 1 + 4 + C = 10$$

$$C = 6$$

12. f, g cont. $f(x) \geq 0$

$$I \int_a^b f g dx = \int_a^b f dx \cdot \int_a^b g dx \quad \text{no.}$$

$$II \int_a^b f + g dx = \int_a^b f dx + \int_a^b g dx \quad \text{yes.} \quad \textcircled{B}$$

$$III \int_a^b \sqrt{f} dx = \sqrt{\int_a^b f dx} \quad \text{no.} \quad \sqrt{F(b) - F(a)}$$

$$13. 2 \cos 3x \quad \frac{2\pi}{3} = \text{fund per} \quad \textcircled{A}$$

$$14. \int \frac{3x^2 dx}{\sqrt{x^3 + 1}} \quad \text{let } u = x^3 + 1$$

$$du = 3x^2 dx$$

$$\int u^{-\frac{1}{2}} du$$

$$2u^{\frac{1}{2}} + C$$

$$2\sqrt{x^3 + 1} + C \quad \textcircled{A}$$

$$15. f(x) = (x-2)^u (x-3)^2$$

$$f'(x) = (x-3)^2 + 2(x-2)(x-3)$$

$$(x-3) [x-3 + 2x-4]$$

$$(x-3)(3x-7)$$

$$f' \begin{array}{c} + \quad 0 \quad - \quad 0 \quad + \\ 0 \quad \frac{7}{3} \quad 3 \end{array}$$

(D) f' test

$$16. y = 2 \ln(\sec x) \quad x = \frac{\pi}{4}$$

$$y' = \frac{2 \sec x \tan x}{\sec x} = 2$$

1 for normal. $\therefore$

$$m = -\frac{1}{2} \quad (B)$$

Not required but worth doing.

$$y|_{x=\frac{\pi}{4}} = 2 \ln \sqrt{2} = \ln 2$$

$$(4, \ln 2)$$

$$y - \ln 2 = -\frac{1}{2}(x - 4)$$

$$y = -\frac{1}{2}x + 2 + \ln 2$$

$$17. \int (x^2 + 1)^2 dx =$$

$$\int x^4 + 2x^2 + 1 dx = \frac{x^5}{5} + \frac{2}{3}x^3 + x + C \quad (E)$$

$$18. f(x) = \sin \frac{x}{2} \quad \frac{\pi}{2} < x < \frac{3\pi}{2}$$

$$f'(x) = \frac{1}{2} \cos \frac{x}{2} \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2} \quad \frac{\pi}{4} < \frac{x}{2} < \frac{3\pi}{4} \quad \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \quad \text{Set } \frac{1}{2} \cos \frac{x}{2}$$

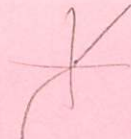
$$\sin \frac{3\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\frac{x}{2} = \frac{\pi}{2} \quad x = \pi$$

$$0 = \cos \frac{x}{2} \quad \therefore \frac{x}{2} = \frac{\pi}{2} \quad x = \pi$$

(D)

19. $f(x) = \begin{cases} x^3 & x \leq 0 \\ x & x > 0 \end{cases}$



a) f is odd $f(-x) = -f(x)$? No, no it is cont at $x=0$

b) f is not cont at $x=0$

c) f has rel max No

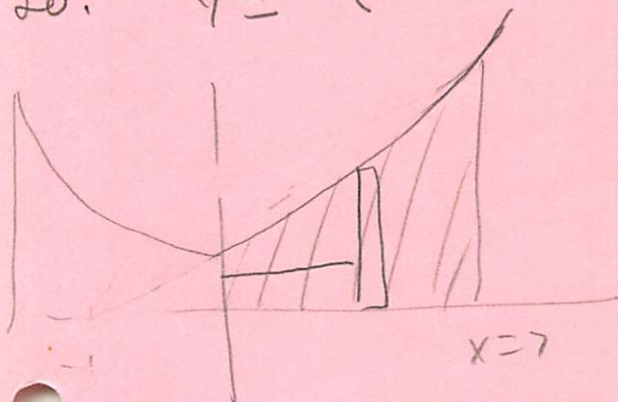
d) $f'(0) = 0$ No $f'(0)$ DNE

e) $f'(x) > 0$ for $x \neq 0$ Yes

$$f' = \begin{cases} 3x^2 & x < 0 \\ 1 & x > 0 \end{cases}$$

(E)

20. $y = (x+1)^{\frac{1}{3}}$ $x=7$



$$2\pi \int_0^7 x (x+1)^{\frac{1}{3}} dx$$

(B)

21.

$$y = \frac{1}{x^2} - \frac{1}{x^3}$$

$$y = x^{-2} - x^{-3}$$

$$y' = -2x^{-3} + 3x^{-4}$$

$$y'' = 6x^{-4} - 12x^{-5}$$

$$6x^{-4} \left(1 - \frac{2}{x}\right)$$

$$\frac{x-2}{x^5}$$

$$\frac{6(x-2)}{x^5}$$

PI?

NOT defined

PI only at

$$y'' + \frac{0}{x} - \frac{0}{x^2} +$$

$x=2$

(C)

22.

$$\int \frac{1}{x^2 - 2x + 2} dx$$

$$\ln^{-1}(x-1) + C \quad (E)$$

$$\int \frac{1}{(x-1)^2 + 1} dx$$

23.

$$f(x) = (x+2)^5 (x-3)^4$$

$$f'(x) = (x-3)^4 \cdot 5(x+2)^4 + (x+2)^5 \cdot 4(x-3)^3$$

$$(x-3)^3 (x+2)^4 \left[(x-3) \cdot 5 + (x+2) \cdot 4 \right]$$

$$(x-3)^3 (x+2)^4 [5x - 15 + 4x + 8]$$

$$(x-3)^3 (x+2)^4 [9x - 7] \quad \text{zeros of } f'$$

$$x \in \{3, -2, \frac{7}{9}\}$$

$$3 \text{ zeros } \therefore 3 \text{ critical points}$$

$$(C)$$

24.

$$f(x) = (x^2 - 2x - 1)^{\frac{2}{3}}$$

$$f'(x) = \frac{2}{3} (x^2 - 2x - 1)^{-\frac{1}{3}} (2x - 2) =$$

$$\frac{2}{3} (0 - 0 - 1) (-2) = x=0 + \frac{4}{3} \quad (A)$$

25.

$$\frac{d}{dx} (2^x) = 2^x \ln 2 \quad (C)$$

26.

$$S(\tau) = -4 \cos \tau - \frac{\tau^2}{2} + 10$$

$$U(\tau) = 4 \sin \tau - \tau$$

$$a(\tau) = 4 \cos \tau - 1$$

$$4 \cos \tau - 1 \stackrel{\text{set}}{=} 0$$

$$\cos \tau = \frac{1}{4} \quad \tau = 1.318$$

$$v(1.318) = 2.555 \quad (D)$$

27. $f(x) = x^3 + 12x - 24$
 $f'(x) = 3x^2 + 12 > 0 \forall x \therefore \textcircled{C}$

28. $\int_1^{500} 13^x - 11^x dx + \int_2^{500} 11^x - 13^x dx$

$\int_1^2 13^x - 11^x dx + \int_2^{500} \underbrace{13^x - 11^x + 11^x - 13^x}_{\text{zero.}} dx$

$\text{FN} \pm \text{NT}(13^x - 11^x, x, 1, 2) = 14.946 \textcircled{B}$

$\int_1^2 e^{x \ln 13} - e^{x \ln 11} dx$

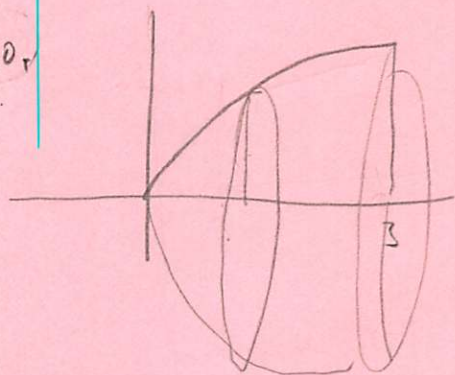
$\left. \frac{1}{\ln 13} 13^x - \frac{1}{\ln 11} 11^x \right|_1^2 = 14.946 \text{ STILL } \textcircled{B}$

$60.8199 - 45.8736$

Note if you use $13^x \ln 13$ as integral (rather than der of 13^x) you get E answer.

29. $\lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{2 \sin^2 \theta} \cdot \frac{1 + \cos \theta}{1 + \cos \theta}$
 $\lim_{\theta \rightarrow 0} \frac{\sin^2 \theta}{2 \sin^2 \theta (1 + \cos \theta)} = \frac{1}{2 \cdot 2} = \frac{1}{4} \textcircled{C}$

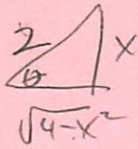
30.



$\pi \int_0^3 x dx$
 $\pi \frac{x^2}{2} \Big|_0^3 = \frac{9\pi}{2} \textcircled{C}$

31. $f(x) = e^{3 \ln x^2}$
 $= e^{\ln x^6}$
 $= x^6 \quad f' = 6x^5 \quad \textcircled{E}$

32. $\int_0^{\sqrt{3}} \frac{dx}{\sqrt{4-x^2}}$



$x = 2 \sin \theta \quad x=0 \quad \theta=0$
 $dx = 2 \cos \theta \, d\theta \quad x = \sqrt{3} \quad \theta = \frac{\pi}{3}$

$\int_0^{\frac{\pi}{3}} \frac{2 \cos \theta \, d\theta}{2 \cos \theta} =$

$\int_0^{\frac{\pi}{3}} d\theta \quad \theta \Big|_0^{\frac{\pi}{3}} = \frac{\pi}{3} \quad \textcircled{A}$

33. $\frac{dy}{dx} = 2y^2 \quad (1, -1) \quad (2, ?)$

$\frac{dy}{y^2} = 2 \, dx$

$-\frac{1}{y} = 2x + 1$

$-\frac{1}{y} = 4 - 1$

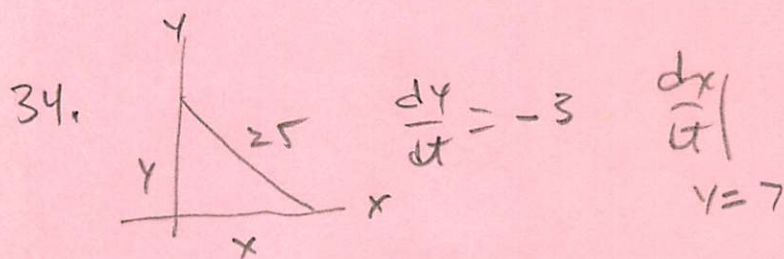
$-\frac{1}{y} = 2x + C$

$-\frac{1}{y} = 3$

$-1 = 2 + C$

$y = -\frac{1}{3} \quad \textcircled{B}$

$C = -1$



$x^2 + y^2 = 25$

$x \frac{dx}{dt} + y \frac{dy}{dt} = 0$

$\frac{dx}{dt} = -\frac{y}{x} \frac{dy}{dt}$

$\frac{dx}{dt} = \frac{3 \cdot 7}{24} = \frac{7}{8} \quad \textcircled{D}$
 $x=24 \quad y=7$

$x^2 + 49 = 625$

$x^2 = 576$

$x = 24$

$$35. \quad y = \frac{ax+b}{x+c}$$

$$\lim_{x \rightarrow \infty} \frac{a + \frac{b}{x}}{1 + \frac{c}{x}} = \frac{a}{1} = 2$$

$$y = 2 \quad x = -3$$

$$\lim_{x \rightarrow \infty} y = 2 \therefore a = 2$$

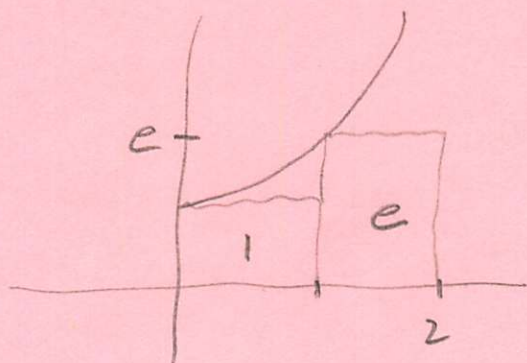
$$x + c = 0$$

$$-3 + c = 0 \therefore c = 3$$

$$a + c = 5$$

(E)

$$36. \quad \int_0^2 e^{x^2} dx$$



TRAP Rule

$$\int_a^b f(x) dx = \frac{b-a}{2n} [f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-1}) + f(x_n)]$$

$$\frac{2}{4} [e^0 + 2e^1 + e^4]$$

$$\frac{1}{2} [1 + 2e + e^4] - (1 + e)$$

$$\frac{1}{2} + e + \frac{1}{2}e^4 - 1 - e \quad \text{from inscribed rectangles.}$$

$$\frac{1}{2}e^4 - \frac{1}{2}$$

$$\frac{1}{2}(e^4 - 1) = 26.7991 \quad \text{(D)}$$

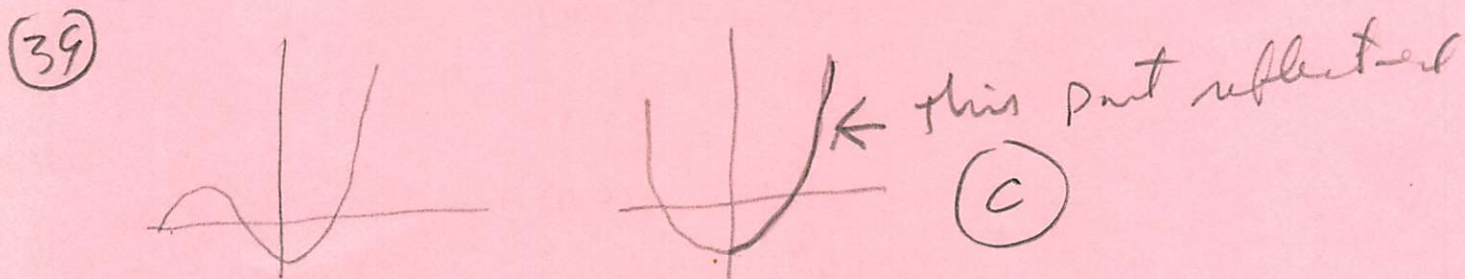
37.

I $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$ 2nd def of deriv at $x=a$

II $\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x-a}$ 1st def of der

I & II (C)

(38) $f'' = 2x - \cos x$
 $f' = x^2 - \sin x$
 $f = \frac{x^3}{3} + \cos x + \dots$ (A)



(40)

$\frac{dn}{dt} > 0$

$\frac{dA}{dt} = \frac{dC}{dt}$

$A = \pi n^2$ $C = 2\pi n$
 $\frac{dA}{dt} = 2\pi n \frac{dn}{dt} \stackrel{\text{set}}{=} \frac{dC}{dt} = 2\pi \frac{dn}{dt}$
 $n = 1$ (D)

(41) $\frac{d}{dx} \int_0^x \cos 2\pi u \, du = \cos 2\pi x$ (D)

(42)

$w|_{t=0} = 2.0$ $w|_{t=2} = 3.5$

$w = A e^{kt}$
 $3.5 = 2 e^{2k}$
 $\sqrt{\frac{7}{4}} = e^k$
 $\frac{\sqrt{7}}{2} = e^k$

$w = 2 \left(\frac{\sqrt{7}}{2} \right)^t$

? $= 2 \left(\frac{\sqrt{7}}{2} \right)^3 = 2 \frac{7\sqrt{7}}{84} = \frac{7\sqrt{7}}{4} \approx 4.63$ (B)

43. $\int x f(x) dx$ Parts

$u = f(x) \quad dv = x dx$

$du = f'(x) dx \quad v = \frac{x^2}{2}$

$\frac{x^2}{2} f(x) - \int \frac{x^2}{2} f'(x) dx$
(B)

44.

$f(x) = x \ln x$

$f'(x) = \ln x + 1 \stackrel{\text{set}}{=} 0$
 $\ln x = -1$
 $x = e^{-1}$

$\frac{-}{0} \quad \frac{0}{e^{-1}} \quad \frac{+}{\text{min } f'(x)}$

$f(e^{-1}) = e^{-1} (\ln e^{-1})$
 $= -\frac{1}{e}$ (C)

45.

$x^3 + x - 1 = 0$

$1 + 1 - 1 = 1$

$3x^2 + 1$

$(1, 1)$

$m = 4$

$\left(\frac{3}{4}\right)^3 + \frac{3}{4} - 1$

$\frac{27}{64} - \frac{16}{64} = \frac{11}{64}$

$y_2 - 1 = 4(x - 1)$

$y_2 - 1 = 4x - 4$

$y_2 = 4x - 3 \stackrel{\text{set}}{=} 0$
 $x = \frac{3}{4}$

$\left(\frac{3}{4}, \frac{11}{64}\right)$

$3 \cdot \frac{9}{16} + 1$

$\frac{27}{16} + \frac{16}{16} = \frac{43}{16} = m$

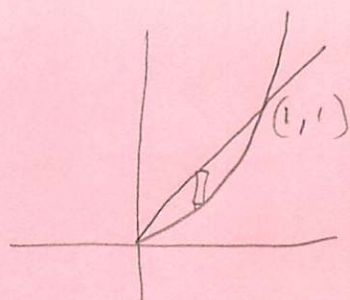
$y - \frac{11}{64} = \frac{43}{16} \left(x - \frac{3}{4}\right)$

$0 - \frac{11}{64} = \frac{43}{16} x - \frac{129}{64}$

$\frac{59}{43} = x$

$\frac{59}{86} = x = .686$ (B)

46.



$$\int_0^1 x - x^2 dx$$

$$\left. \frac{x^2}{2} - \frac{x^3}{3} \right|_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6} \quad (A)$$

47. $f(x) = 2x^2 + 1$

$$\lim_{x \rightarrow 0} \frac{f(x) - f(0)}{x^2}$$

$$\lim_{x \rightarrow 0} \frac{2x^2 + 1 - 1}{x^2} = 2 \quad (C)$$

48. $Q(x) = \int_0^x P(t) dt$

degree of P is N so degree of Q is $N+1$ E

49. $xy = 10 \quad x = 2 \quad \frac{dy}{dt} = 3$

$$y \frac{dx}{dt} + x \frac{dy}{dt} = 0$$

$$5 \frac{dx}{dt} + 2(3) = 0$$

$$\frac{dx}{dt} = -\frac{6}{5} \quad (B)$$

50. $r = 2 \sec \theta$

$$r \cos \theta = 2$$

$$x = 2$$

D

51.

$$x = t^2 + 1$$

$$\frac{dx}{dt} = 2t$$

$$y = t^3$$

$$\frac{dy}{dt} = 3t^2$$

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$$

$$= 3t^2 \cdot \frac{1}{2t}$$

$$\frac{dy}{dx} = \frac{3}{2} t$$

$$\frac{d^2y}{dx^2} = \frac{3}{2} \frac{dt}{dx}$$

$$\frac{3}{2} \cdot \frac{1}{2t}$$

A

$$52. \int_0^1 x^3 e^{x^4} dx$$

$$u = x^4 \quad x=0 \quad u=0 \\ du = 4x^3 dx \quad x=1 \quad u=1$$

$$\frac{1}{4} \int_0^1 e^u du = \frac{e^u}{4} \Big|_0^1 = \frac{1}{4}(e-1) \quad (A)$$

$$53. f(x) = \ln e^{2x} \\ = 2x \quad f'(x) = 2 \quad (B)$$

$$54. f(x) = 1 + x^{\frac{2}{3}}$$

$$f'(x) = \frac{2}{3} x^{-\frac{1}{3}}$$

$$f''(x) = -\frac{2}{9} x^{-\frac{4}{3}}$$

- a) f is cont No Dom Rest. True
 b) there is a min at $x=0$ True
 c) f is increasing when $x > 0$ True

- d) f' DNE at $x=0$ False. (D)
 e) $f'' < 0$ $\forall x > 0$ True.

$$55. \ln x \text{ is cont at } x=1 \\ e^x \text{ is cont at } x=1 \\ \ln(e^x - 1) \text{ is cont at } x=1$$

I, II, and III

(e)

$$56. \int_4^{\infty} \frac{-2x dx}{\sqrt[3]{9-x^2}}$$

$$\text{let } u^3 = 9-x^2 \\ 3u^2 du = -2x dx$$

$$x \rightarrow 4 \quad u \rightarrow \sqrt[3]{5} \\ x \rightarrow \infty \quad u \rightarrow -\infty$$

$$\lim_{b \rightarrow \infty} \int_{\sqrt[3]{5}}^b \frac{3u^2 du}{u}$$

$$\frac{3u^2}{2} \Big|_{\sqrt[3]{5}}^b$$

$$\lim_{b \rightarrow \infty} \frac{3}{2} \left[b^2 - \sqrt[3]{5}^2 \right] \rightarrow +\infty$$

Does not exist.

(E)

57. $x(t) = \sin 2t - \cos 3t$ $a(\pi) = ?$

$v(t) = 2 \cos 2t + 3 \sin 3t$

$a(t) = -4 \sin 2t + 9 \cos 3t$ $t = \pi$ $-4(0) + 9(-1) = -9$ (E)

58. $\frac{dy}{dx} = x^2 y$ $y = C_2 e^{\frac{x^3}{3} + C}$ (C)

$\frac{dy}{y} = x^2 dx$

$\ln y = \frac{x^3}{3} + C$

59. $f' = x^4(x-2)(x+3)$ $+0-0-0+$ -3 0 2 f'
 max only at $x = -3$ one (B)

60. $f(x) = e^{\tan^2 x}$

$f'(x) = e^{\tan^2 x} 2 \tan x \sec^2 x$ (D)

61. a mean value problem $\int_a^b f(x) = f(c)(b-a)$ (E)

62. $\ln(xy) = x$ $x=1$ $\ln y = 1$

$\frac{1}{xy} \left[y + x \frac{dy}{dx} \right] = 1$ $(1, e)$ $y = e$

$y + x \frac{dy}{dx} = xy$

$\frac{dy}{dx} = \frac{xy - y}{x}$ $\frac{1 \cdot e - e}{1} = 0$ (A)

63.

$$e^{f(x)} = 1+x^2$$

or

$$\ln e^{f(x)} = \ln 1+x^2$$

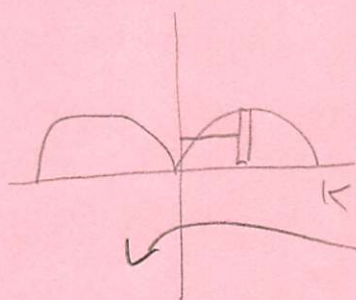
$$f(x) = \ln 1+x^2$$

$$f'(x) = \frac{2x}{1+x^2}$$

$$e^{f(x)} \cdot f'(x) = 2x$$

$$f'(x) = \frac{2x}{e^{f(x)}} = \frac{2x}{1+x^2} \quad (13)$$

64.



$$2\pi \int_0^k x(kx-x^2) dx \stackrel{\text{set}}{=} 10$$

$$\int_0^k kx^2 - x^3 dx = \frac{5}{\pi}$$

$$\frac{k^4}{3} - \frac{k^4}{4} = \frac{5}{\pi}$$

$$\frac{k^4}{12} = \frac{5}{\pi}$$

$$k = \sqrt[4]{\frac{60}{\pi}} = 2.0905 \quad (13)$$

65.

$$a(\tau) = e^{-2\tau}$$

$$v(0) = \frac{5}{2}$$

$$x(0) = \frac{17}{4}$$

$$v(\tau) = -\frac{1}{2}e^{-2\tau} + c$$

$$-\frac{1}{2}e^{-2 \cdot 0} + c = \frac{5}{2}$$

$$-\frac{1}{2} + c = \frac{5}{2}$$

$$c = \frac{4}{2} = 2$$

$$x(\tau) = \int -\frac{1}{2}e^{-2\tau} + 3 d\tau$$

$$= +\frac{1}{4}e^{-2\tau} + 3\tau + c$$

$$x(0) = +\frac{1}{4} + 0 + c = \frac{17}{4}$$

$$c = \frac{16}{4} = 4$$

$$x(\tau) = \frac{1}{4}e^{-2\tau} + 3\tau + 4$$

(E)

66.

$$y = \frac{\sqrt[3]{x^2+8}}{\sqrt[4]{2x+1}}$$

$$y' = \frac{\frac{2x}{3}(x^2+8)^{-\frac{2}{3}} - (x^2+8)^{\frac{1}{3}} \cdot \frac{2}{4}(2x+1)^{-\frac{3}{4}}}{(2x+1)^{\frac{1}{2}}}$$

$$\frac{1 \cdot 0 \cdot \frac{1}{4} - 2 \cdot \frac{1}{2}}{1} = -1 \quad (A)$$

67. $f(x) = x^2 e^x$
 $f'(x) = 2xe^x + x^2 e^x$
 $x e^x (2 + x)$ $\frac{+0-0+}{-2 \quad 0}$ (B)

68. $\text{length} = \int_a^b \sqrt{\dot{x}^2 + \dot{y}^2} dt$
 $x = t^2 \quad y = t$
 $\dot{x} = 2t \quad \dot{y} = 1$
 $\int_0^4 \sqrt{4t^2 + 1} dt$ (D)

69. $\lim_{x \rightarrow 0} \frac{f(x)}{g(x)} = \frac{0}{0}$ l'Hopital's rule

$\lim_{x \rightarrow 0} \frac{f'(x)}{g'(x)}$ (C)

70. $x = e^t \quad y = t e^{-t}$ $\left. \frac{dy}{dx} \right|_{x=2} = ?$

$\frac{dx}{dt} = e^t = x$ $\frac{dy}{dt} = e^{-t} - t e^{-t}$
 $t = \ln x$ $= \frac{1-t}{e^t}$

$\frac{1 - \ln x}{x} \cdot \frac{1}{x}$
 $\frac{1 - \ln x}{x^2} = \frac{1 - \ln 3}{9}$
 $- .01095$ (D)

71. $y = \tan^{-1}(e^{2x})$
 $y' = \frac{1}{1+e^{4x}} \cdot 2e^{2x}$
 $y' = \frac{2e^{2x}}{1+e^{4x}}$ (B)

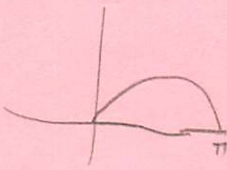
72. $\int x \sec^2 x dx$ "Parts" = $x \tan x - \int \tan x dx$ $\frac{\sin}{\cos}$
 $u = x$ $dv = \sec^2 x dx$
 $du = dx$ $v = \tan x$
 $x \tan x + \ln |\cos x| + C$
 (E)

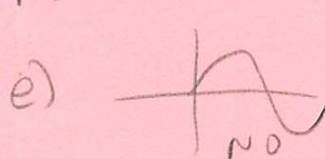
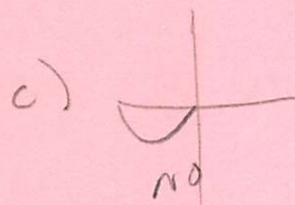
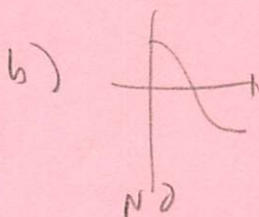
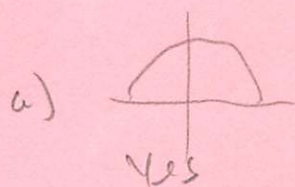
73.  $y = \sec x$
 $\pi \int \sec^2 x dx$
 $\pi \int_0^{\pi/3} \sec^2 x dx$
 $\pi \tan x \Big|_0^{\pi/3}$
 $\pi \sqrt{3}$ (C)

74. $\int_a^b f(x) dx = 5$ $\int_a^b g(x) dx = -7$ II only
 (B)

I $f > g$? Not necessarily

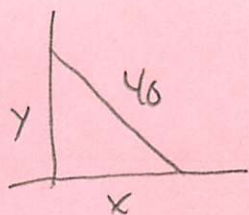
II Yes, Sum of Parts is Sum of whole.
 III No Product doesn't work for integral

75. $\int_0^{\pi} \sin x dx$ 



(A)

76,



$$x^2 + y^2 = 40^2 \quad \text{find } x \text{ when } \frac{dx}{dt} = \frac{3}{4} \frac{dy}{dt}$$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

$$x = -y \frac{dy}{dt} \cdot \frac{dt}{dx}$$

$$= -y \frac{dy}{dt} \cdot \frac{4}{3} \frac{dt}{dy}$$

$$x = -\frac{4}{3} y \quad y = -\frac{3}{4} x$$

$$y^2 = \frac{9}{16} x^2$$

$$x^2 + \frac{9}{16} x^2 = 40^2$$

$$\frac{25}{16} x^2 = 40^2$$

$$x^2 = 40^2 \cdot \frac{16}{25}$$

$$= 32 \quad \text{(E)}$$

$$77, \quad F'(x) = \int_0^x f(t) dt \quad F'(a) = -2 \quad F'(b) = -2 \quad a < b$$

$$\int_0^a f(t) dt + \int_a^b f(t) dt = \int_0^b f(t) dt$$

$$F'(a) + \int_a^b f(t) dt = F'(b)$$

$$-2 + \int_a^b f(t) dt = -2$$

$$\int_a^b f(t) dt = 0$$

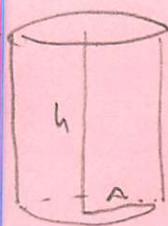
since $a \neq b$

then $f(t) \equiv 0$.

so for some x $f(x) = 0$.

(A)

78,



$$2\pi r + h = 30, \quad V = \pi r^2 h$$

$$h = 30 - 2\pi r$$

$$V(r) = \pi r^2 (30 - 2\pi r)$$

$$= 30\pi r^2 - 2\pi^2 r^3$$

$$V' = 60\pi r - 6\pi^2 r^2$$

$$6\pi r(10 - \pi r) \stackrel{\text{set}}{=} 0$$

$$V \quad - \quad 0 \quad + \quad 0 \quad -$$

$$\quad \quad \quad \uparrow \quad \quad \quad \uparrow$$

$$\quad \quad \quad r=0 \quad \quad \quad \frac{10}{\pi}$$

max at $\frac{10}{\pi}$ $f'(6.5)$. (E)

79,

$$f(x) = \begin{cases} x & \text{if } x \leq 1 \\ \frac{1}{x} & \text{if } x > 1 \end{cases}$$

$$\int_0^e f(x) dx =$$

$$\int_0^1 f(x) dx + \int_1^e f(x) dx$$

$$\int_0^1 x dx + \int_1^e \frac{1}{x} dx$$

$$\left. \frac{x^2}{2} \right|_0^1 + \left. \ln x \right|_1^e$$

$$\rightarrow \frac{1}{2} + \ln e$$

$$\frac{1}{2} + 1 = \frac{3}{2}. \quad (B)$$

80,

$$f(t) = Ae^{kt}$$

$$1200 = 1000e^{7k}$$

$$\left(\frac{6}{5}\right)^{\frac{1}{7}} = e^k$$

$$f(t) = 1000 \left(\frac{6}{5}\right)^{\frac{t}{7}}$$

$$= 1000 \left(\frac{6}{5}\right)^{\frac{12}{7}}$$

$$= 1366.907$$

1367 people.

(C)

(81)

$$\frac{dy}{dx} = \frac{1}{x}$$

$$\int dy = \frac{1}{3} \int \frac{1}{x} dx$$

"average
value"

$$\frac{1}{3} \ln x \Big|_1^4$$

$$\frac{1}{3} \ln 4 = \frac{1}{3} \ln 2^2$$

$$\underline{\underline{\frac{2}{3} \ln 2}} \quad (C)$$

(82)

$$y = \ln(1 + 2x - x^2)$$

$$\ln(1) = 0 \text{ so}$$

$$1 + 2x - x^2 = 1 \text{ set}$$

$$2x - x^2 = 0$$

$$x(2-x) = 0$$

$$x \in \{0, 2\}$$

Simpson's Rule,

$$\frac{h}{3} [f(x_0) + 4f(x_1) + 2f(x_2) + \dots + f(x_n)]$$

$$x_0 = 0 \quad h = 1$$

$$x_1 = 1 \quad f(x_0) = f(x_n) = 0$$

$$x_2 = 2$$

$$\frac{1}{3} [0 + 4 \cdot \ln(2)]$$

$$\frac{4}{3} \ln 2 = .924196$$

$$(C)$$

(83)

$$f(x) = \int_{-2}^{x^2-3x} e^{t^2} dt$$

$$f'(x) = e^{(x^2-3x)^2} \cdot (2x-3) \stackrel{\text{set } 0}{=} 0$$

$$\therefore x = \frac{3}{2}$$

$$(C)$$

(84)

$$\lim_{x \rightarrow 0} (1+2x)^{\csc x}$$

$$\lim_{x \rightarrow 0} (1+2x)^{\frac{1}{\sin x}}$$

$$\lim_{x \rightarrow 0} \frac{(1+2x)^{\frac{1}{2x} \cdot 2 \cdot \frac{x}{\sin x}}}{e^{2 \cdot 1}}$$

$$e^2 \quad (E)$$